On local structure of cubicity 2 graphs

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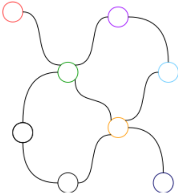
COCOA 2016

** With Sujoy Bhore, Sandip Das, Sagnik Sen.

Cubicity 2 graphs

Intersection graph of unit squares on the plane



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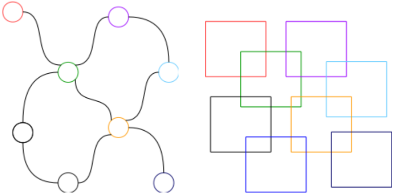

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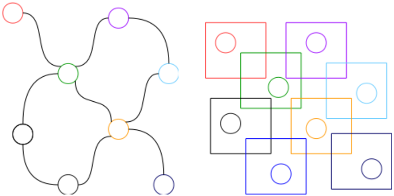

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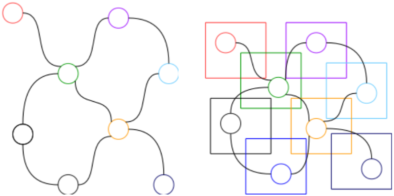

Cubicity 2 graphs

Unfortunately, deciding whether an input graph is a cubicity 2 graph, is NP-hard.

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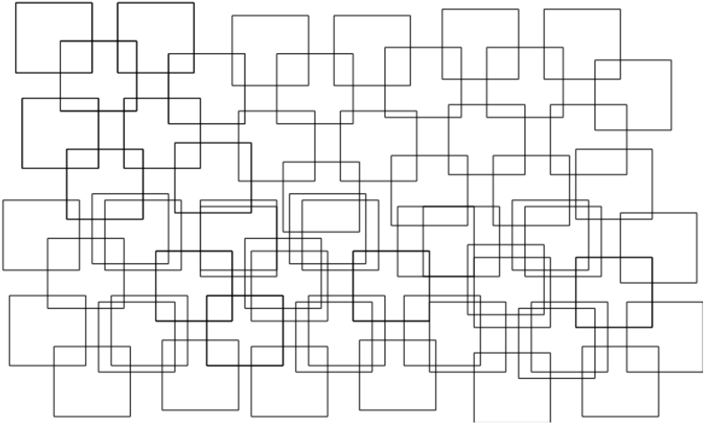
In fact, the complexity of deciding whether an input tree is a cubicity 2 graph, is unknown. (LS Chandran 2014 [1])

Today we shall solve the above problem "locally".

[1] Babu, Jasine, Manu Basavaraju, L. Sunil Chandran, Deepak Rajendraprasad, and Naveen Sivadasan."Approximating the cubicity of trees." arXiv preprint (2014).

Local structure of cubicity 2 graphs



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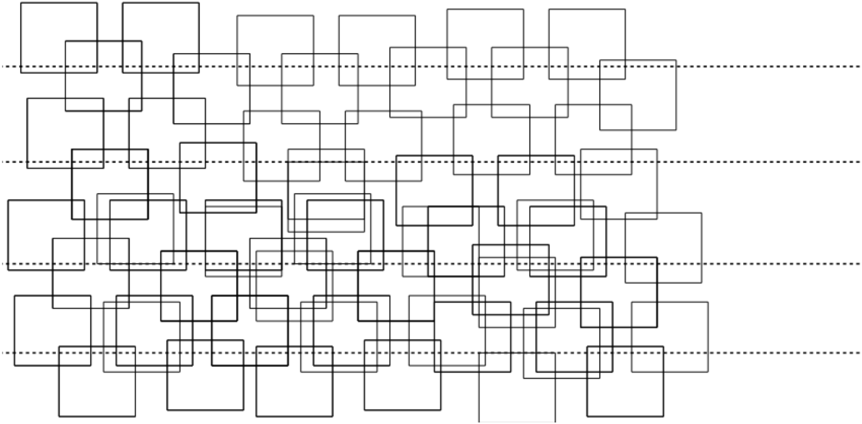

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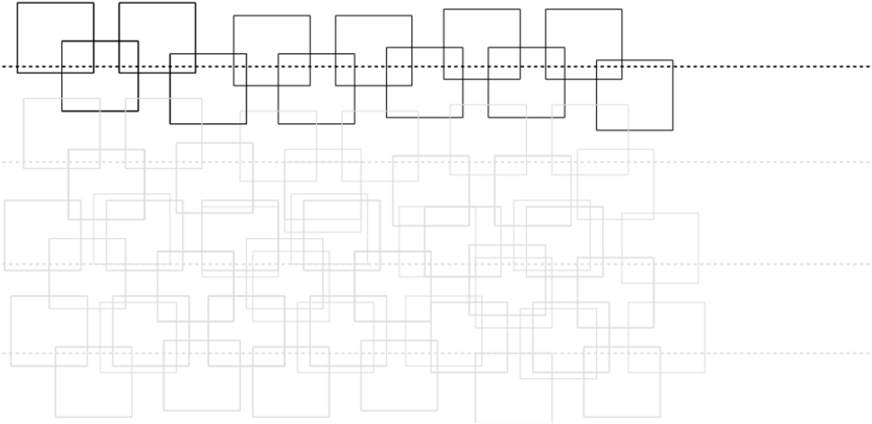

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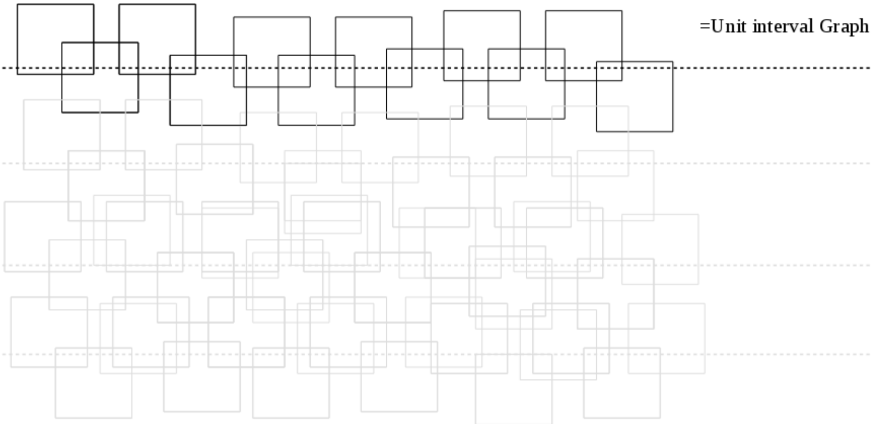

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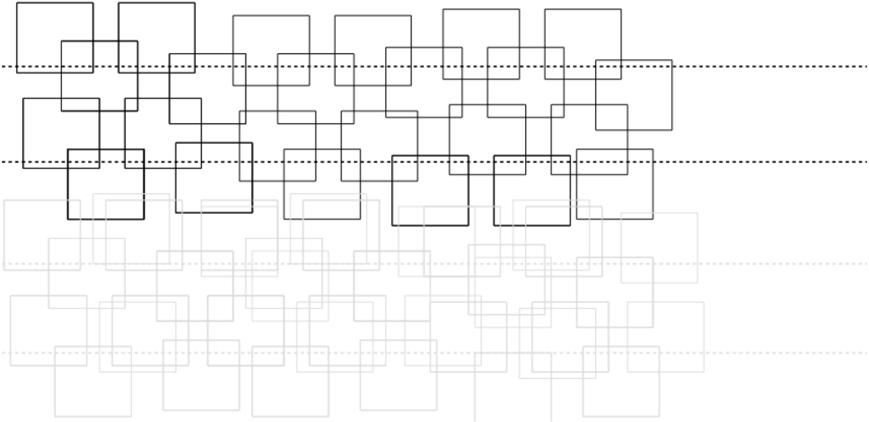

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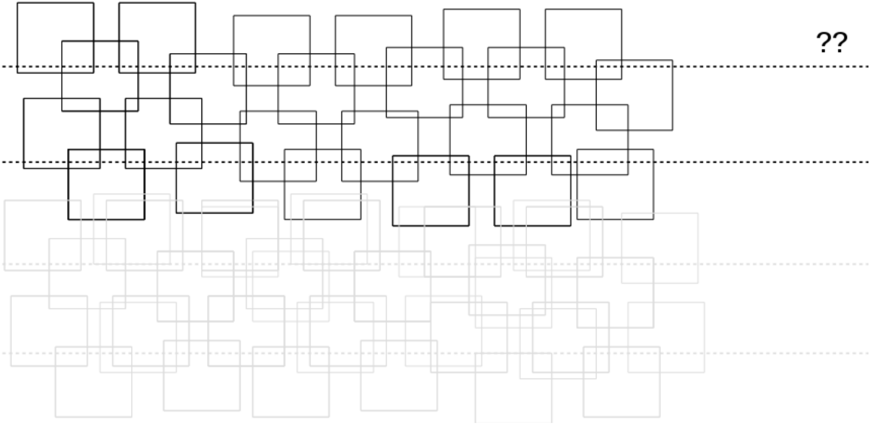

2SUIG graphs (2-stab unit interval graph)

A graph is a 2SUIG graph if it has intersection representation such that

- Two stab lines in $1 + \epsilon$ distance apart.

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A graph is a 2SUIG graph if it has intersection representation such that

- Two stab lines in $1+\epsilon$ distance apart.
- Axes-parallel unit squares.
- Each unit square intersects exactly one stab line.

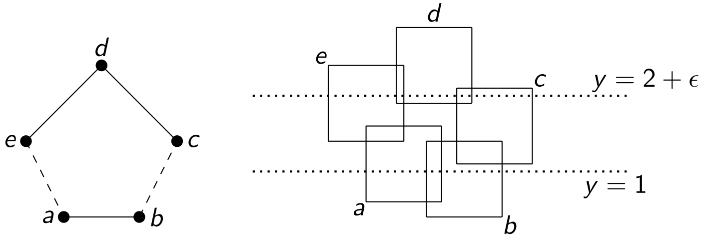

Figure: A representation (right) of a 2*SUIG* graph (left).

So, let us try to recognize Tree $\cap$ 2SUIG $=$ Tree-2SUIG graphs

In fact we prove that $\cdots$

Tree-2SUIG recognition Algorithm: Overview

Theorem

There is a $O(n)$ time algorithm to recognize tree-2SUIG graphs.

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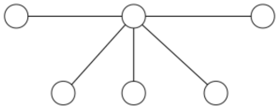
If the input tree satisfies all those conditions, output a 2SUIG representation of the tree.

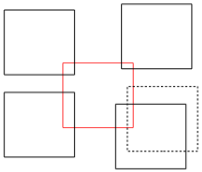
Otherwise, output a subtree that violets a particular condition.

Tree-2SUIG recognition Algorithm: Overview

Lemma

If a tree T has a 2SUIG representation then $\Delta(T) \leq 4$, $\Delta(T)$ is the maximum degree of T.





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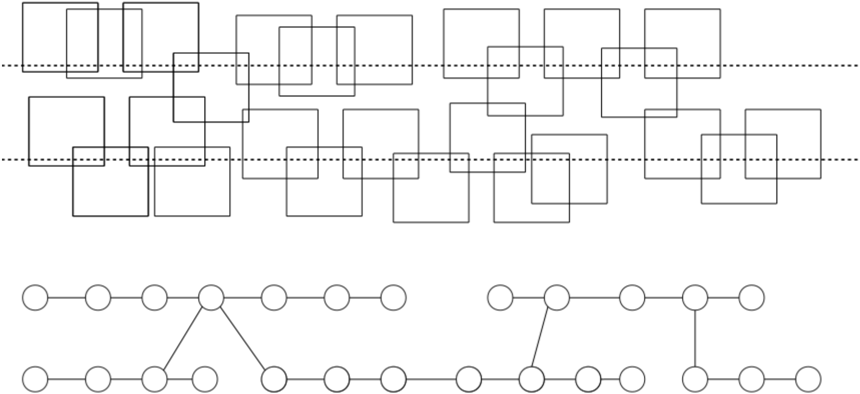

Figure: A typical tree-2SUIG.

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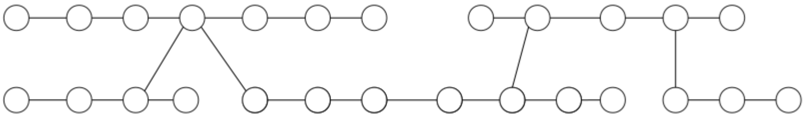

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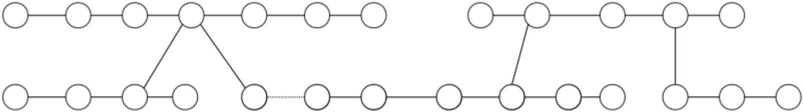
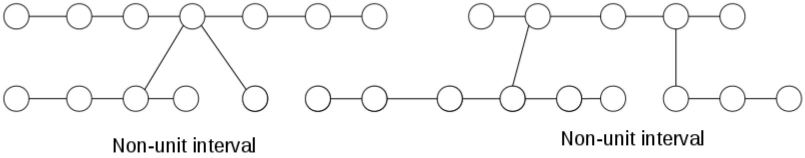


Take an edge and delete it.

Tree-2SUIG recognition Algorithm: Overview



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If both sub trees are non-unit interval,

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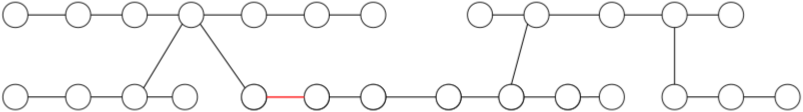

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If both sub trees are non-unit interval, call that edge as *red edge*.

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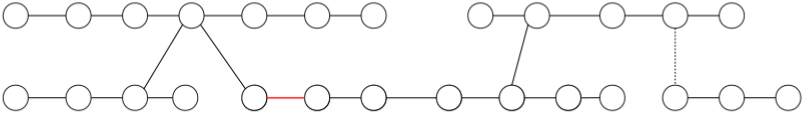

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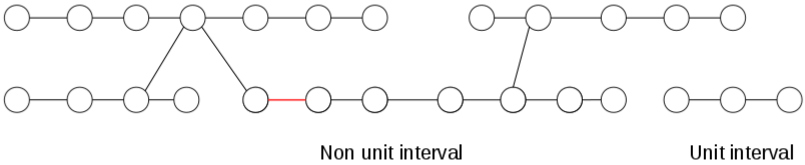

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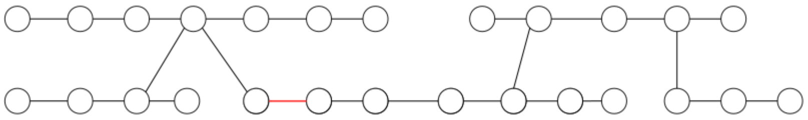

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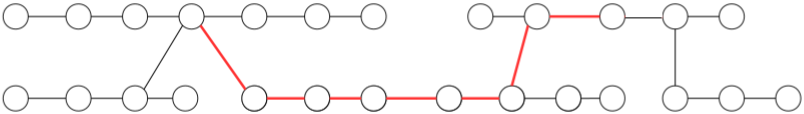

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In this case, do nothing.

Get the set of all red edges.

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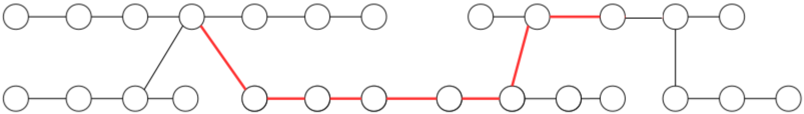

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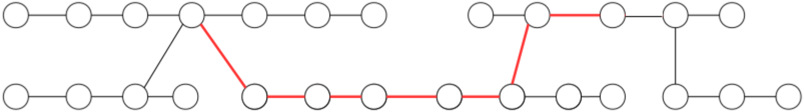
Lemma

If a tree T has a 2SUIG representation, the set of red edges induces a connected path.

Tree-2SUIG recognition Algorithm: Overview



Delete the vertices incident to a red edge.

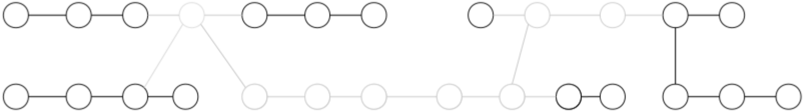


Figure: Only paths are left.

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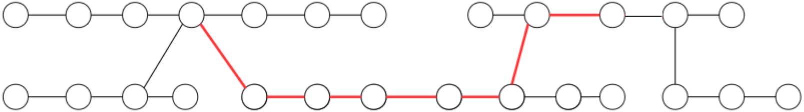

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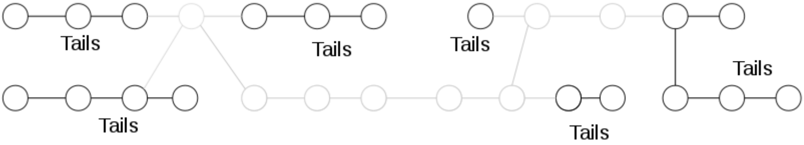

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Tree-2SUIG recognition Algorithm: Overview

Assuming there are red edges,

Represent the red path appropriately.

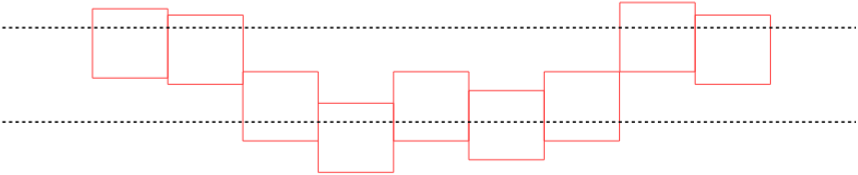


Figure: Stretched representation of red path.

Tree-2SUIG recognition Algorithm: Overview

Assuming there are red edges,

Represent the red path (Stretched representation).

Represent the tails appropriately (Shrinked representation).

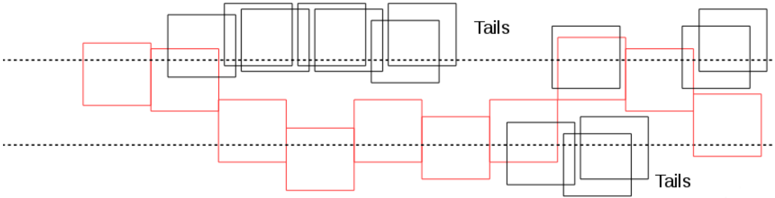


Figure: Shrinked representation of tails.

Length of the tails imposes restriction.

Tree-2SUIG recognition Algorithm: Overview

Assuming there are red edges,

Stretched representation of the red path.

$$+$$

Shrinked representation of the tails.

$$\Downarrow$$

Canonical representation of tree-2SUIG.

Tree-2SUIG recognition Algorithm: Overview

Idea is similar, even when there are no red edges.

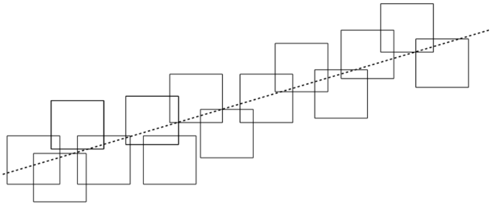
Theorem

There is a $O(n)$ time algorithm to recognize tree-2SUIG graphs.

Open problems

1. Determine the complexity of recognizing trees with cubicity 2.

2. Recognize those graphs whose intersection representation consists of squares, intersecting a common line, not necessarily axes parallel.



3. Recognize 2SUIG graphs.

Thank You