On geodetic sets of graphs

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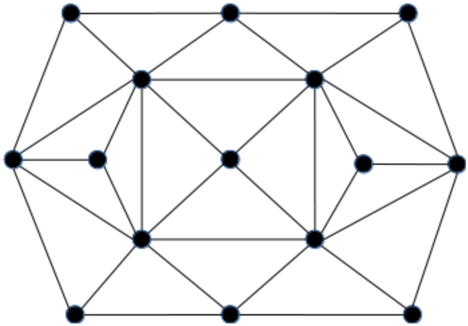
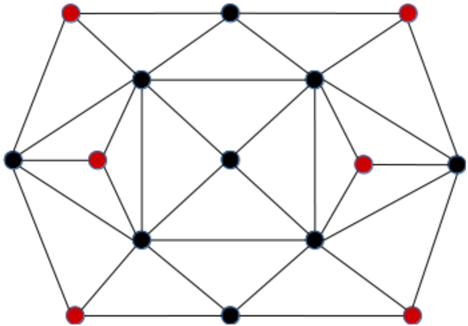
***Joint work with Florent Foucaud, Harmender Gahlawat, Subir Kumar Ghosh, and Bodhayan Roy.

A simplified practical problem



City road network

A simplified practical problem



City road network

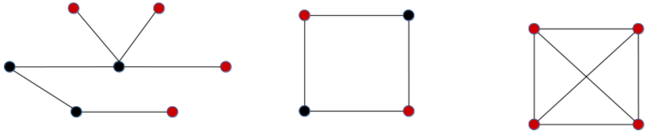
- Open minimum number of new bus terminals such that all other cities are "covered".
- Assumption: Buses follow the shortest path between the terminals.

Definition

A set of vertices S of a graph G is a *geodetic set* if each vertex in $V(G) \setminus S$ lies in some shortest path between some pair of vertices in S.

Minimum Geodetic Set

INPUT: An undirected graph G.
OUTPUT: A geodetic set of minimum cardinality.



Examples

Computing conundrum

Simplified practical problem = Hard theoretical problem.

Why I got interested in this topic

Algorithmic results before 2018

NP-hard: Chordal graphs, bipartite graphs, chordal bipartite graphs, co-bipartite graphs.

Polynomial time algorithms: Distance hereditary graphs, split graphs, ptolemaic graphs, outer-planar graphs, proper interval graphs.

Probably many (easy) open questions !! 🙂

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Important questions

Question 1: Minimum Geodetic Set problem on planar graphs.

Question 2: Minimum Geodetic Set problem on interval graphs.

I started to work on the above from 2017.

Indo-French Meet 2019 @ (ISI, RKMVERI)

Thanks to AGROHO (MA/IFCAM/18/39) and HOSIGRA (ANR-17-CE40-0022) projects.

Éric Sopena , Théo Pierron, Florent Foucaud, Julien Bensmail.

Subir Kumar Ghosh, Sandip Das, Sagnik Sen, Bodhayan Roy, Joydeep Mukherjee, Uma kant Sahoo, D.C, Harmendar, Shubhadeep Ranjan Dev, Sanjana Dey, Arun Kumar Das.

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Hardness and Approximation for the Geodetic Set Problem in Some Graph Classes. D.C., Florent Foucaud, Harmender Gahlawat, Subir Kumar Ghosh, Bodhayan Roy: CALDAM 2020.

Our results

- NP-hardness for planar graphs and line graphs.
- $O(\log n)$-hard on graphs with an universal vertex.
- $O(n^{\frac{1}{3}} \log n)$-approximation algorithm on general graphs.
- 3-approximation algorithm on solid grids.

Some proof ideas !!

- NP-hardness for planar graphs.

▶ Reduction from DOMINATING SET of planar graphs.

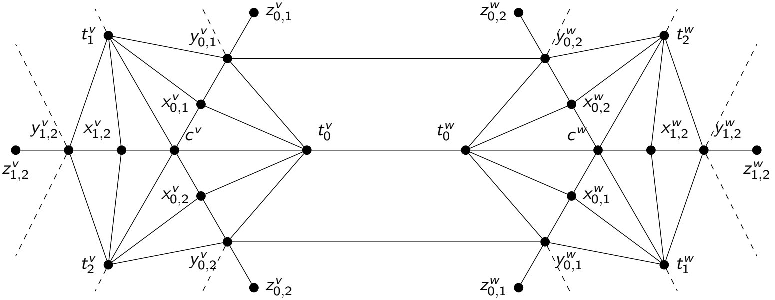

Figure: Here, two vertex gadgets G_v, G_w are depicted, with v and w adjacent in G. Dashed lines represent potential edges to other vertex-gadgets.

Some proof ideas !!

- $O(n^{\frac{1}{3}} \log n)$-approximation algorithm on general graphs.

- Reduction to Minimum Rainbow Subgraph problem.

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Minimum Rainbow Subgraph

INPUT: An undirected edge-colored graph G.
OUTPUT: A subgraph containing edges of all colors with minimum number of vertices.

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Minimum Rainbow Subgraph

INPUT: An undirected edge-colored graph G.
OUTPUT: A subgraph containing edges of all colors with minimum number of vertices.

- Given a graph G, construct an edge-colored graph H as follows.

- If w lies in some shortest path between u, v in G, then put an edge having color "w" between u, v in H.

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Minimum Rainbow Subgraph

INPUT: An undirected edge-colored graph G.
OUTPUT: A subgraph containing edges of all colors with minimum number of vertices.

▶ Given a graph G, construct an edge-colored graph H as follows.

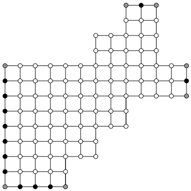
▶ If w lies in some shortest path between u, v in G, then put an edge having color "w" between u, v in H.

▶ Apply known $O(n^{\frac{1}{3}} \log n)$-approximation algorithm on Minimum Rainbow Subgraph problem.

*On the Approximability of the Minimum Rainbow Subgraph Problem and Other Related Problems. Sumedh Tirodkar, Sundar Vishwanathan. ISAAC 2015.

Some proof ideas !!

- 3-approximation algorithm on solid grids.



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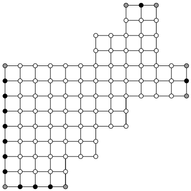

- A path P of a solid grid is a corner path if (i) no vertex of P is a cut vertex, (ii) both end-vertices of P have degree 2, and (iii) all vertices except the end-vertices of P have degree 3.

Some proof ideas !!

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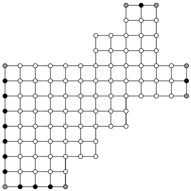

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Lemma: Any geodetic set must contain (i) all pendent vertices and (ii) at least one vertex from each corner path.

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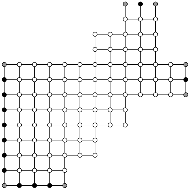

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Lemma: Any geodetic set must contain (i) all pendent vertices and (ii) at least one vertex from each corner path.

Our approximation algorithm: Select all pendent vertices and both end-vertices of each corner path.

The future

- Polynomial time optimal algorithms on solid grids.
- Polynomial time optimal algorithms on partial grids.
- Polynomial time optimal algorithms on interval graphs.
- FPT or approximation algorithms on planar graphs.
- FPT or approximation algorithms on chordal graphs.
- Approximation algorithms on diameter 2 graphs.

The future

- Polynomial time optimal algorithms on solid grids. (Done)
- Polynomial time optimal algorithms on partial grids. (Impossible*)
- Polynomial time optimal algorithms on interval graphs. (Impossible*)

* Assuming P$\neq$ NP.

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- FPT or approximation algorithms on planar graphs ?
- FPT or approximation algorithms on chordal graphs ?
- Approximation algorithms on diameter 2 graphs ?

Thank you !!