Triangle-free projective planar graphs with diameter 2: domination and characterization

Dibyayan Chakraborty, Sandip Das, Srijit Mukherjee, Uma kant Sahoo, Sagnik Sen

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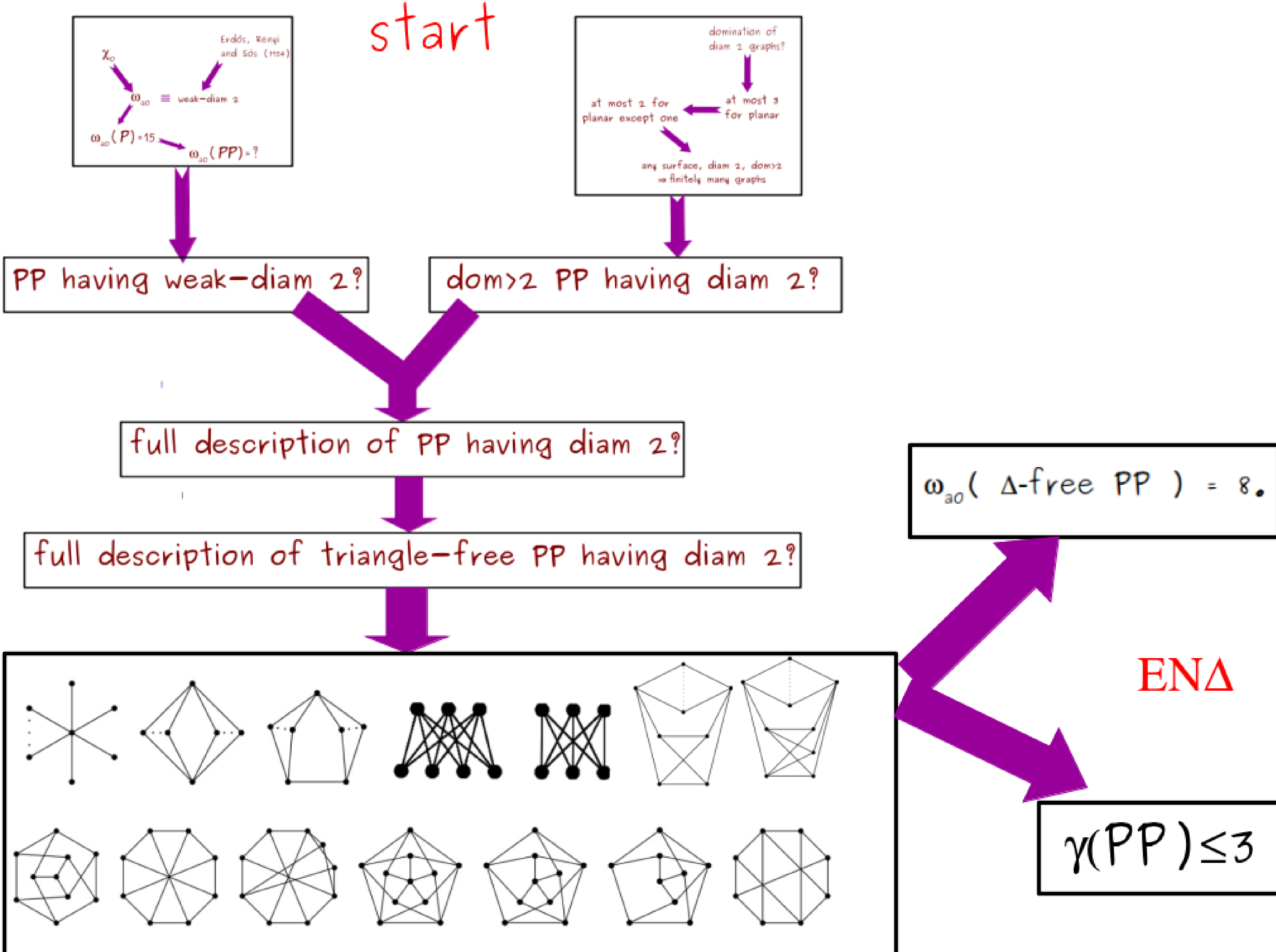
Lyon, France

12 July, 2018



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Erdős, Rényi and Sós (1954)
χo
ωao ≡ weak-diam 2
ωao(P)=15
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ωao(Δ-free PP) = 8.
ENΔ
γ(PP)≤3

Oriented chromatic number (χ_o)

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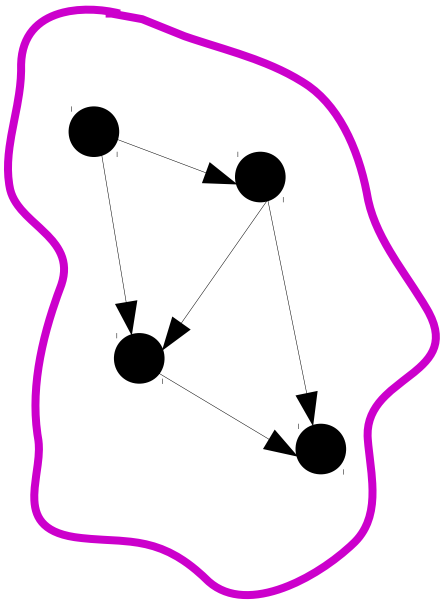

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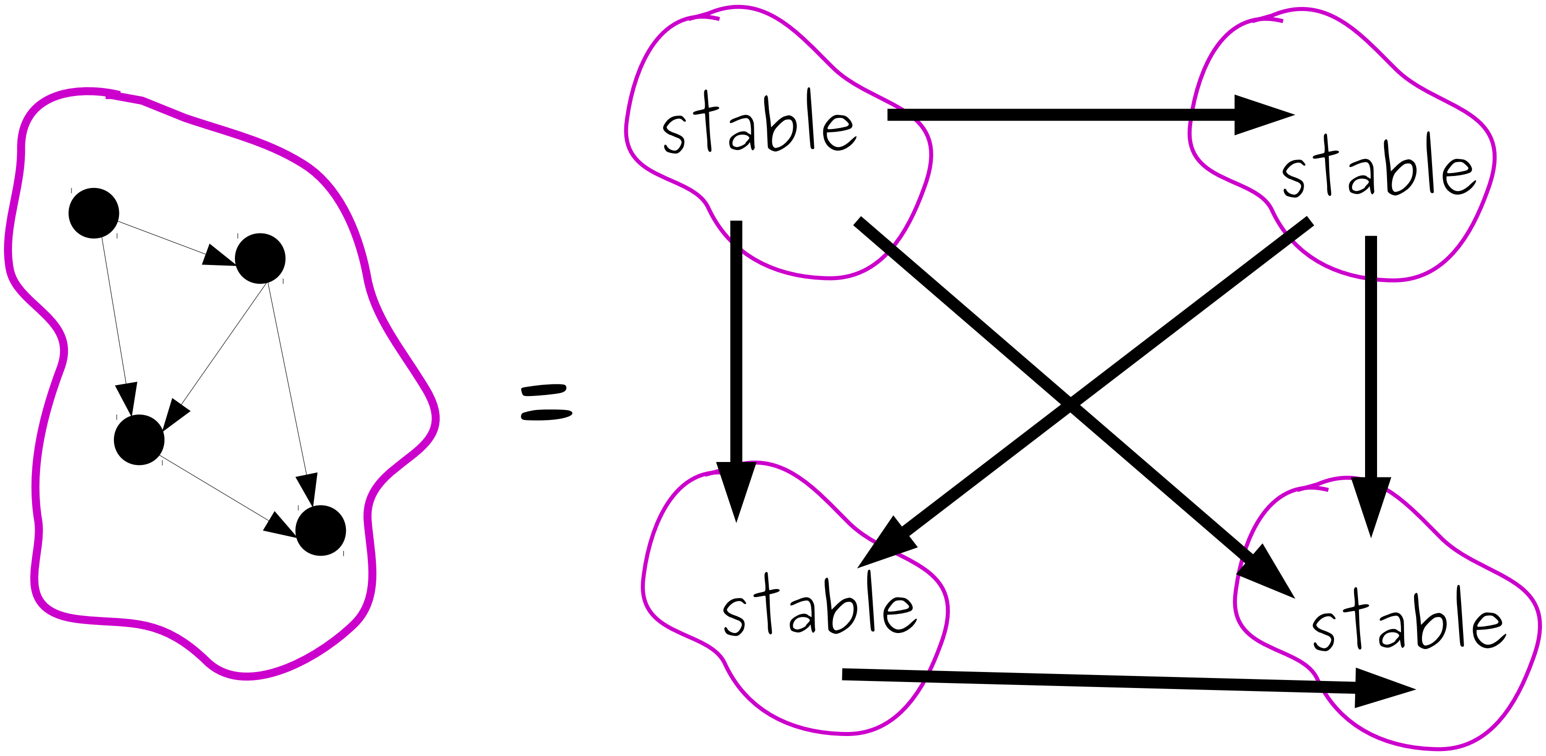

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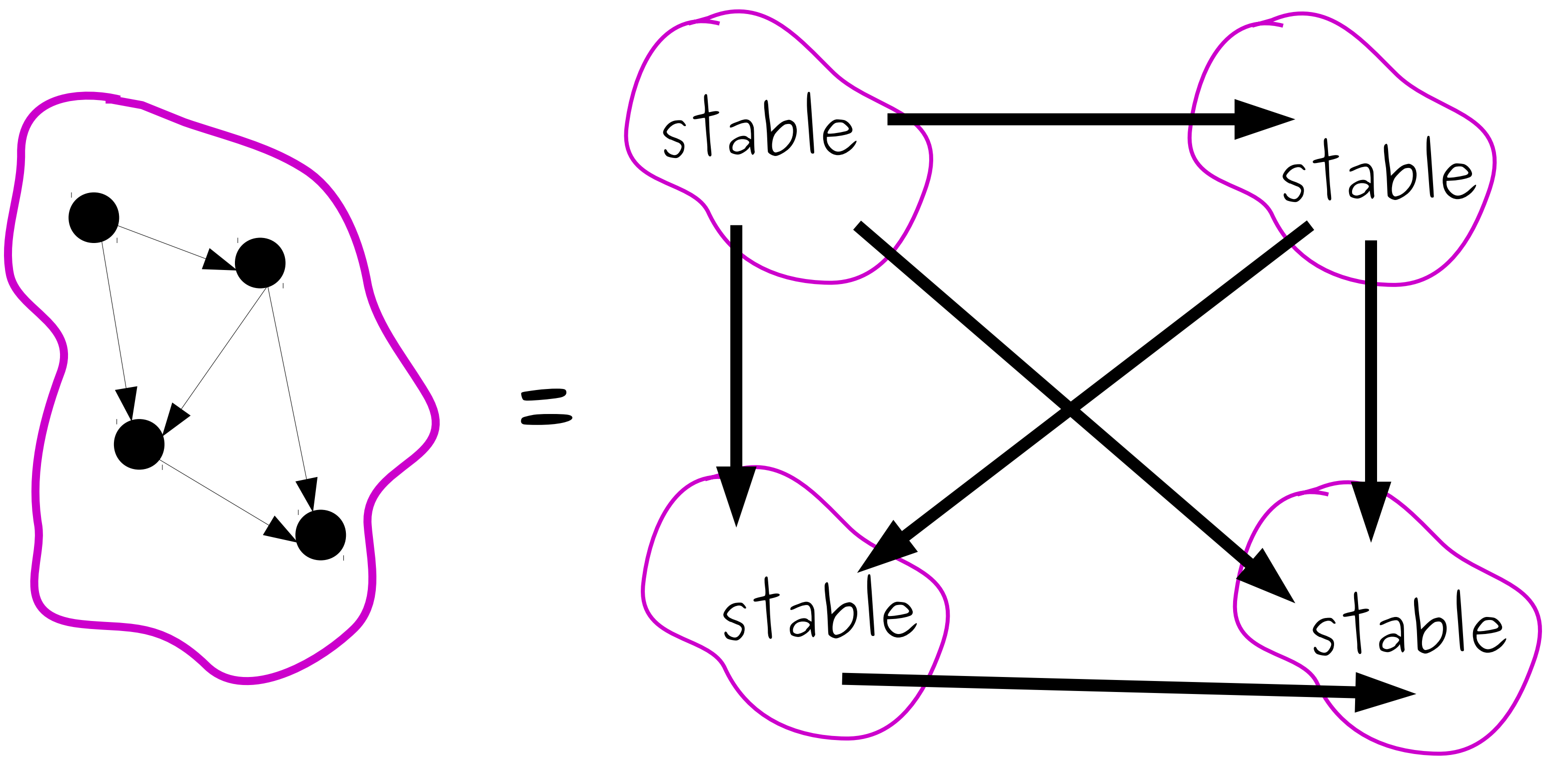
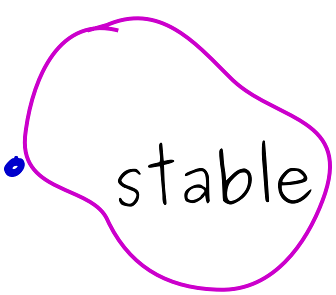


$\chi_o(G) =$ min no. 

χ_o

χ_o ✔

Oriented clique number (ω_{ao})

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O is an oriented clique if $\chi_o(O) = |V(O)|$

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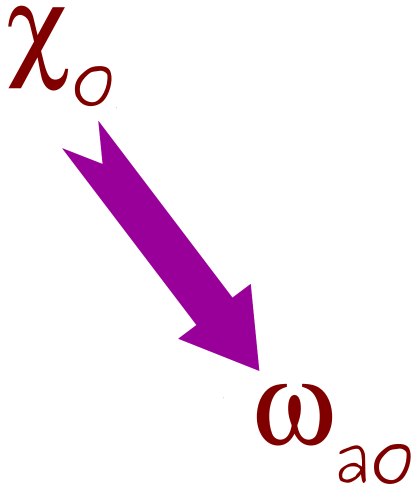
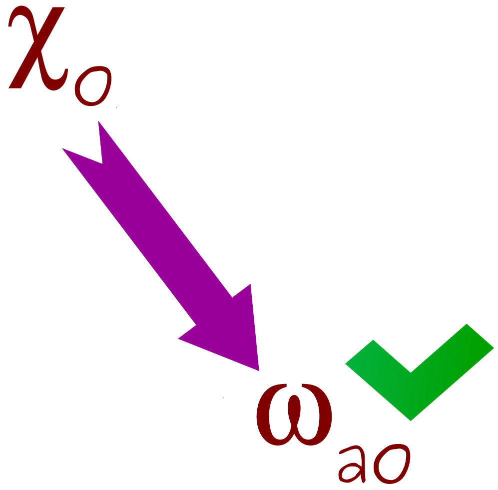
$$\omega_{ao}(G) = \max\{|V(O)| : O \subseteq G\}$$


χ_0
ω_{20}


χ_o
ω_{ao}

Weak diameter 2 (weak-diam 2)

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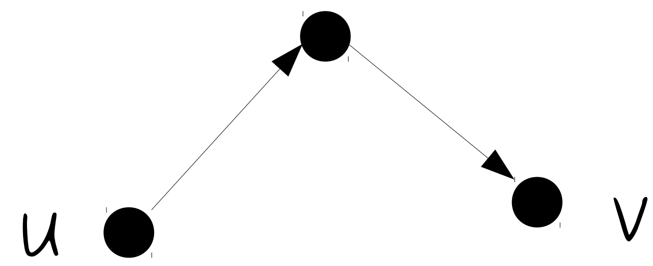
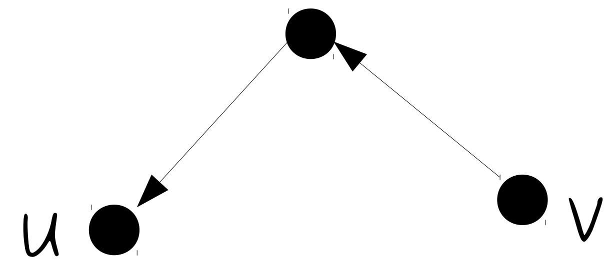
u,v non-adjacent $\Rightarrow$

Weak diameter 2 (weak-diam 2)

u,v non-adjacent ⇒



or



Weak diameter 2 (weak-diam 2)

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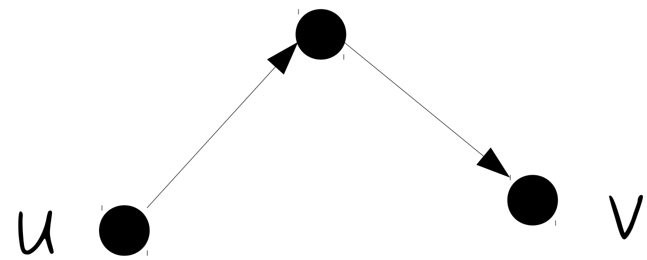

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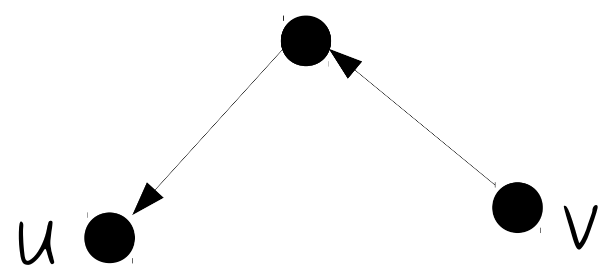
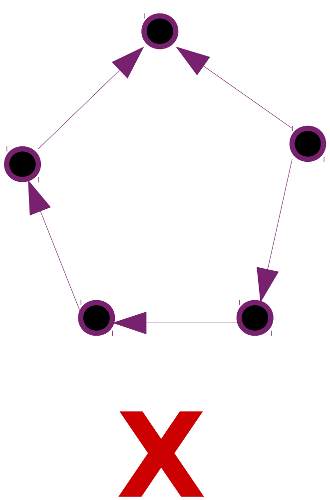
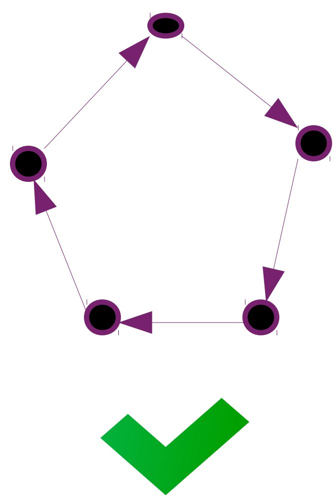






χ_0

$\downarrow$

ω_{a0} weak-diam 2

χ_0

↓

ω_{a0}

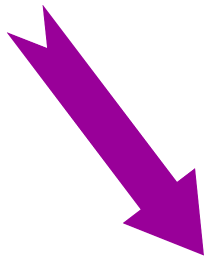
weak-diam 2 ✓

Klostermeyer and MacGillivray (2004)

χ_o

$\omega_{ao} \equiv$ weak-diam 2

χ_0



$\omega_{a0} \equiv$ weak-diam 2

χ_o

Erdős, Rényi and Sós (1954)

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χ_o

$\omega_{ao} \equiv$ weak-diam 2

Katona,
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Horak,
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Zhu,
Kostochka,
Luczak,
Simonyi,
Sopena,
Kirgizov,
Duvignan,
Bensmail

Erdős, Rényi and Sós (1954)

χ_o

$\omega_{ao} \equiv$ weak-diam 2

$\omega_{ao}(P) = 15$

Conjecture (Klostermeyer and MacGillivray 2004)

Erdős, Rényi and Sós (1954)

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Theorem (Nandy, Sen Sopena 2016)

χ_o

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Other surfaces? (Oriol Serra)

Erdős, Rényi and Sós (1954)

χ_o

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~~Other surfaces?~~

Projective plane?

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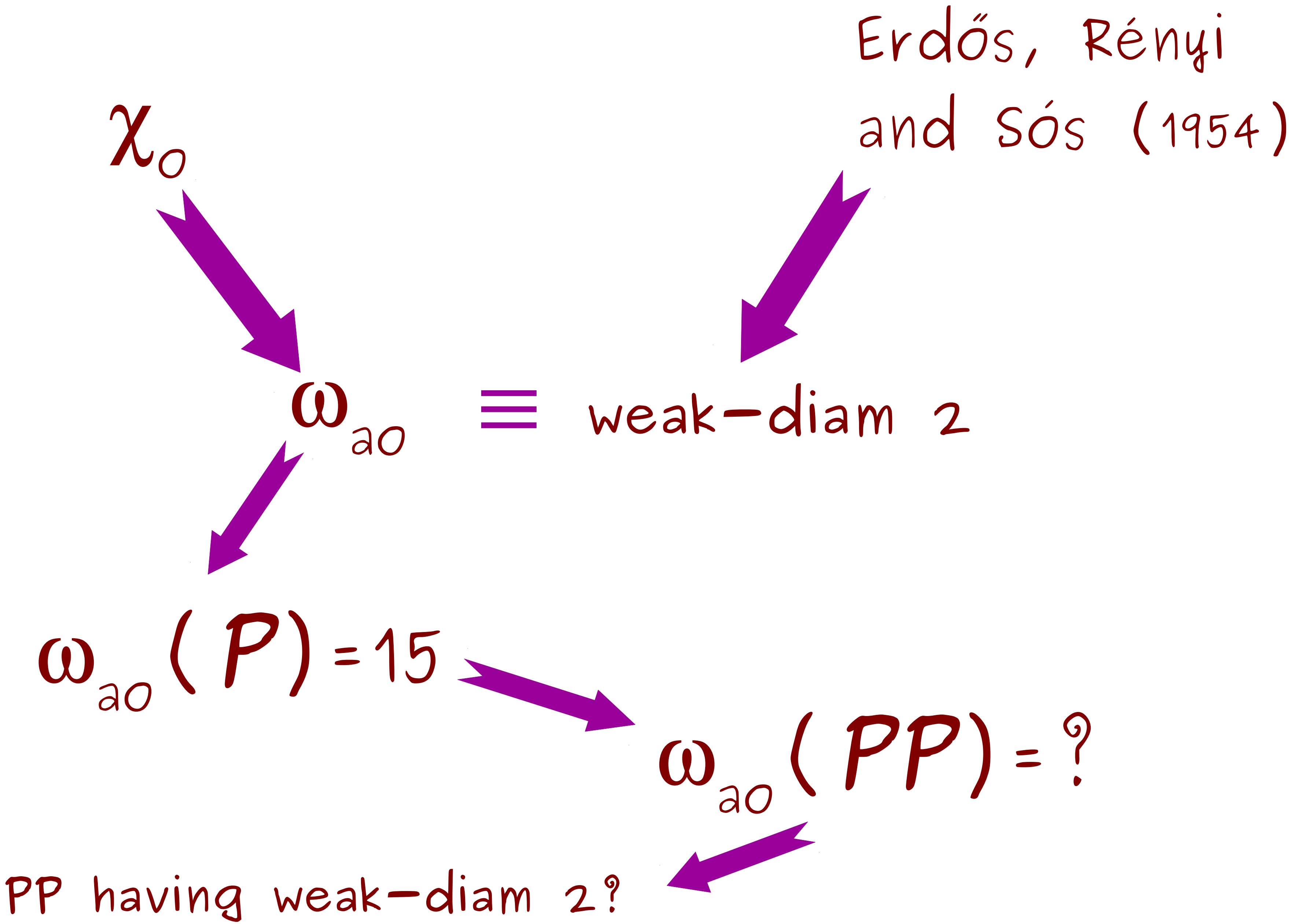

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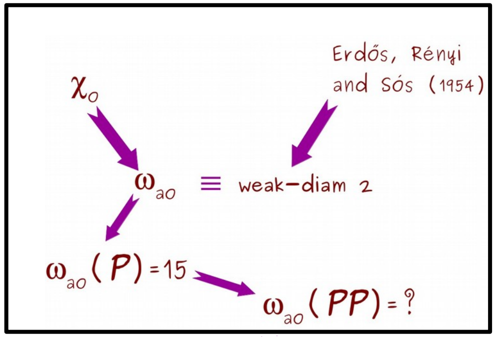

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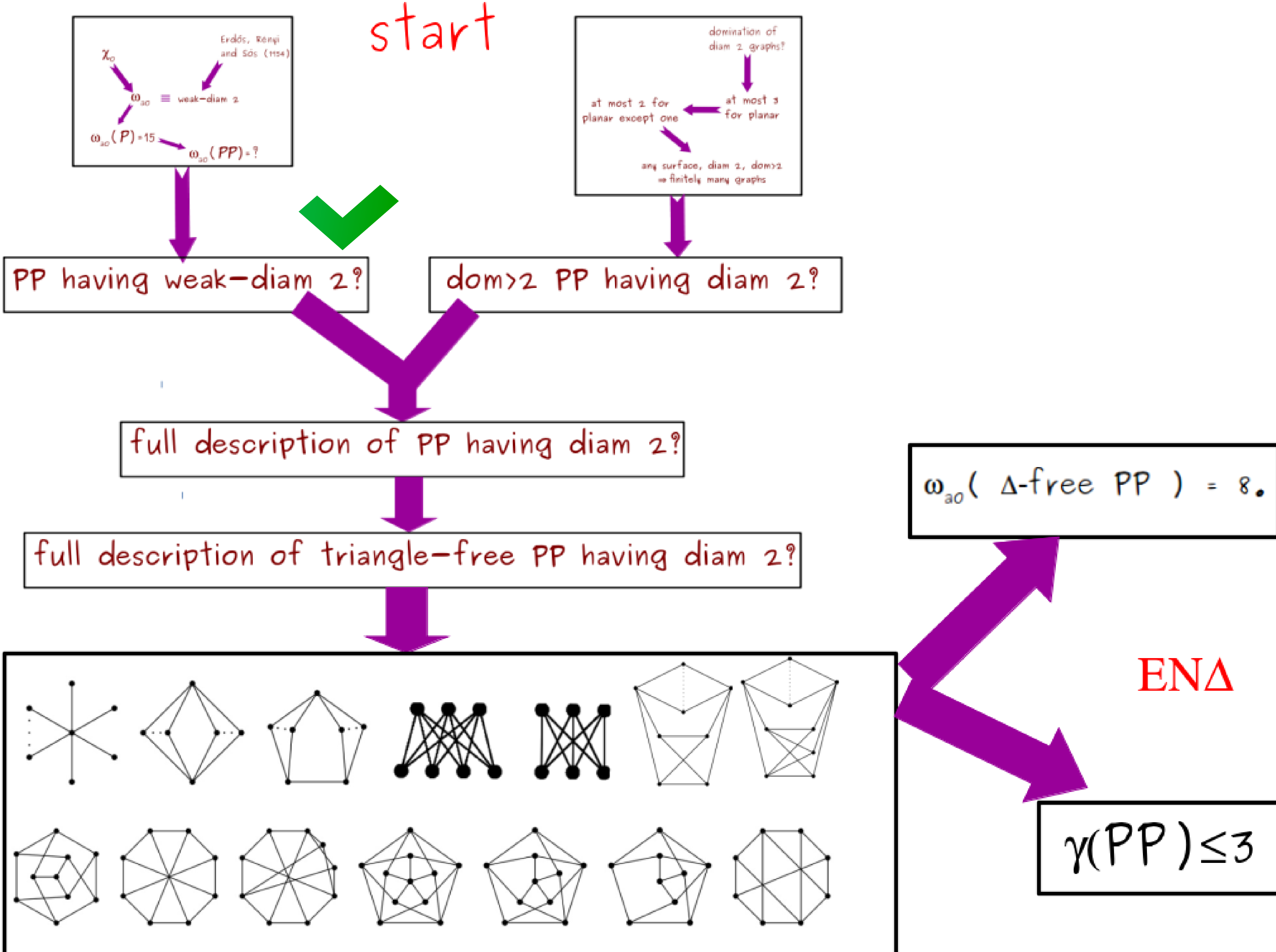
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MacGillivray and
Seyffarth (1996)

at most 3
for planar

domination of
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at most 3
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Goddard and Henning (2002)

domination of diam 2 graphs?

at most 3 for planar

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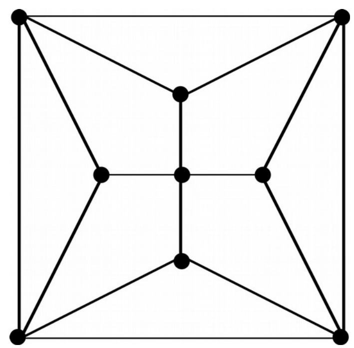

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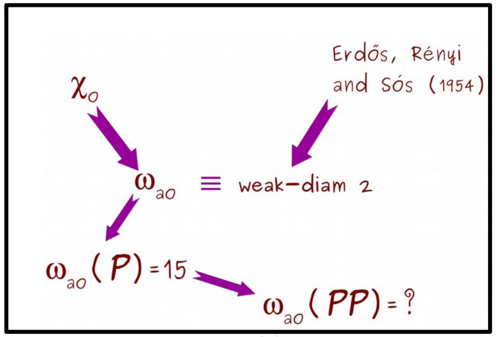
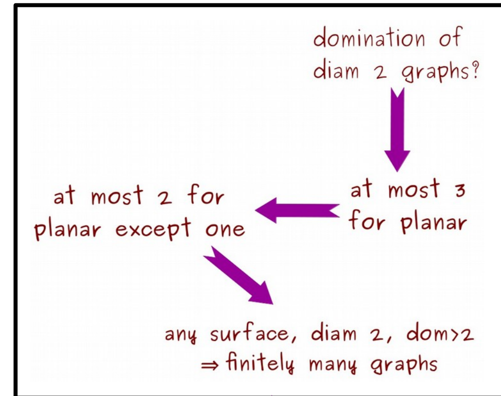
any surface, diam 2, dom>2
$\Rightarrow$ finitely many graphs

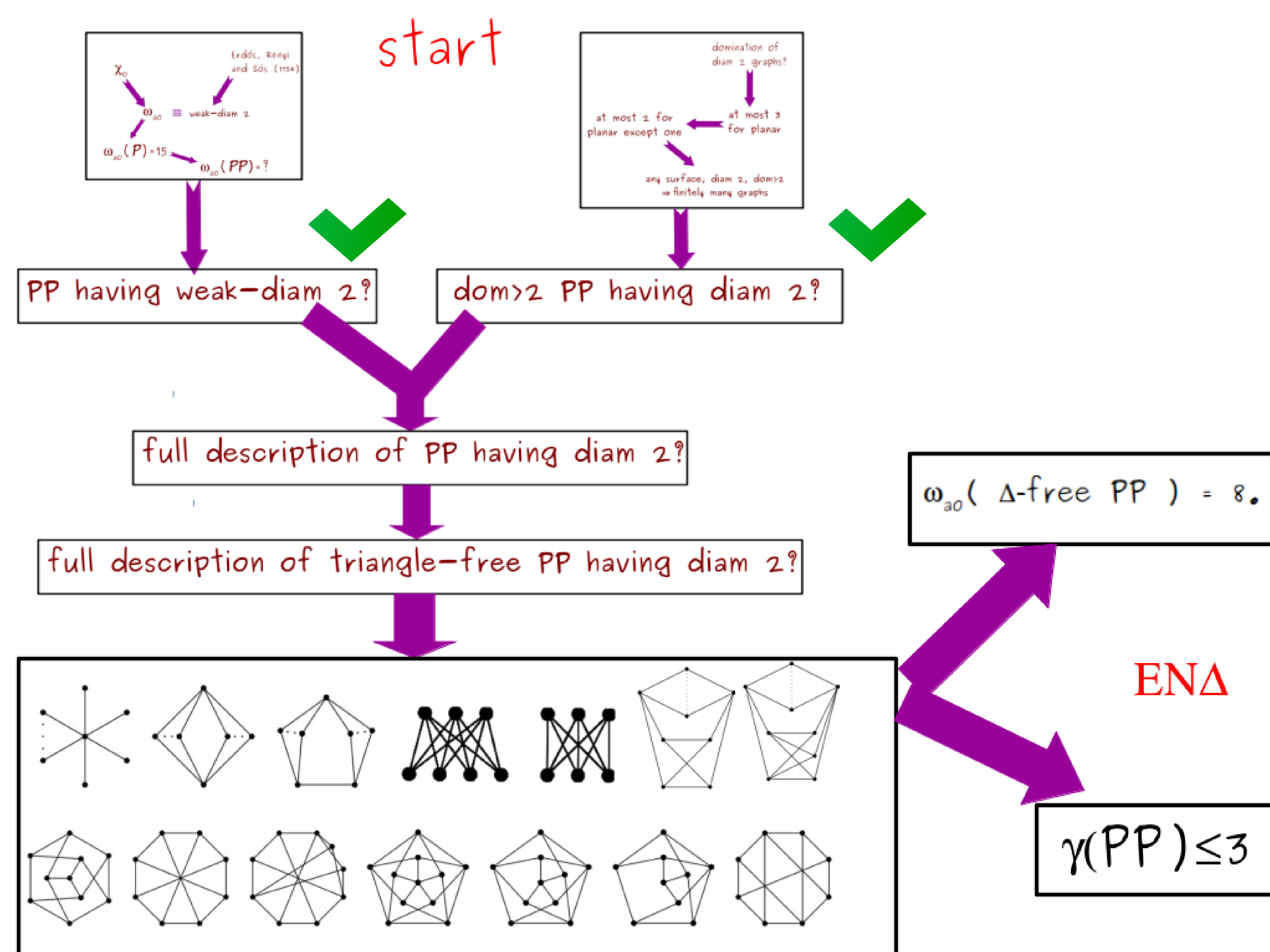
exceptions
in PP?



PP having weak-diam 2?



dom>2 PP having diam 2?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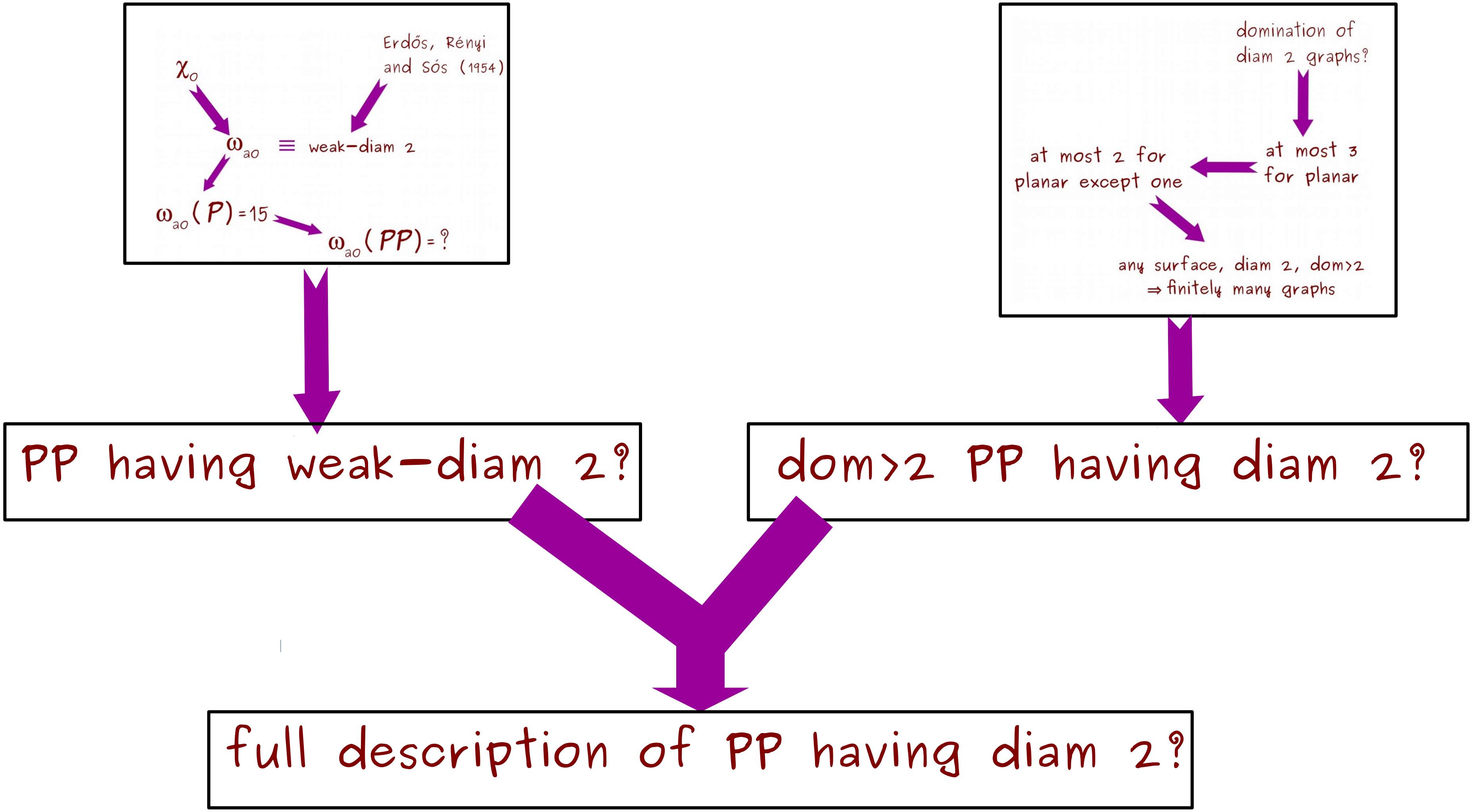

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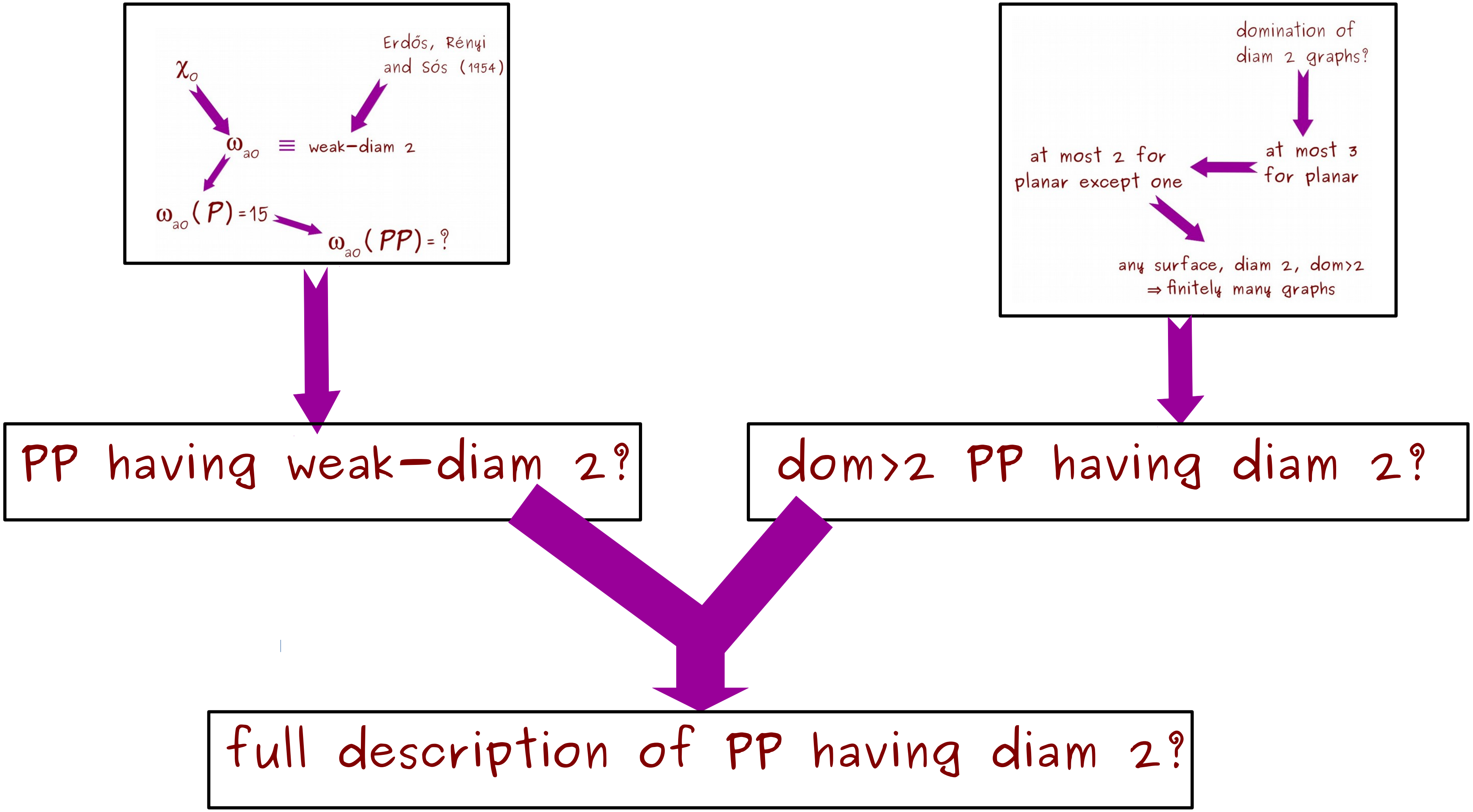
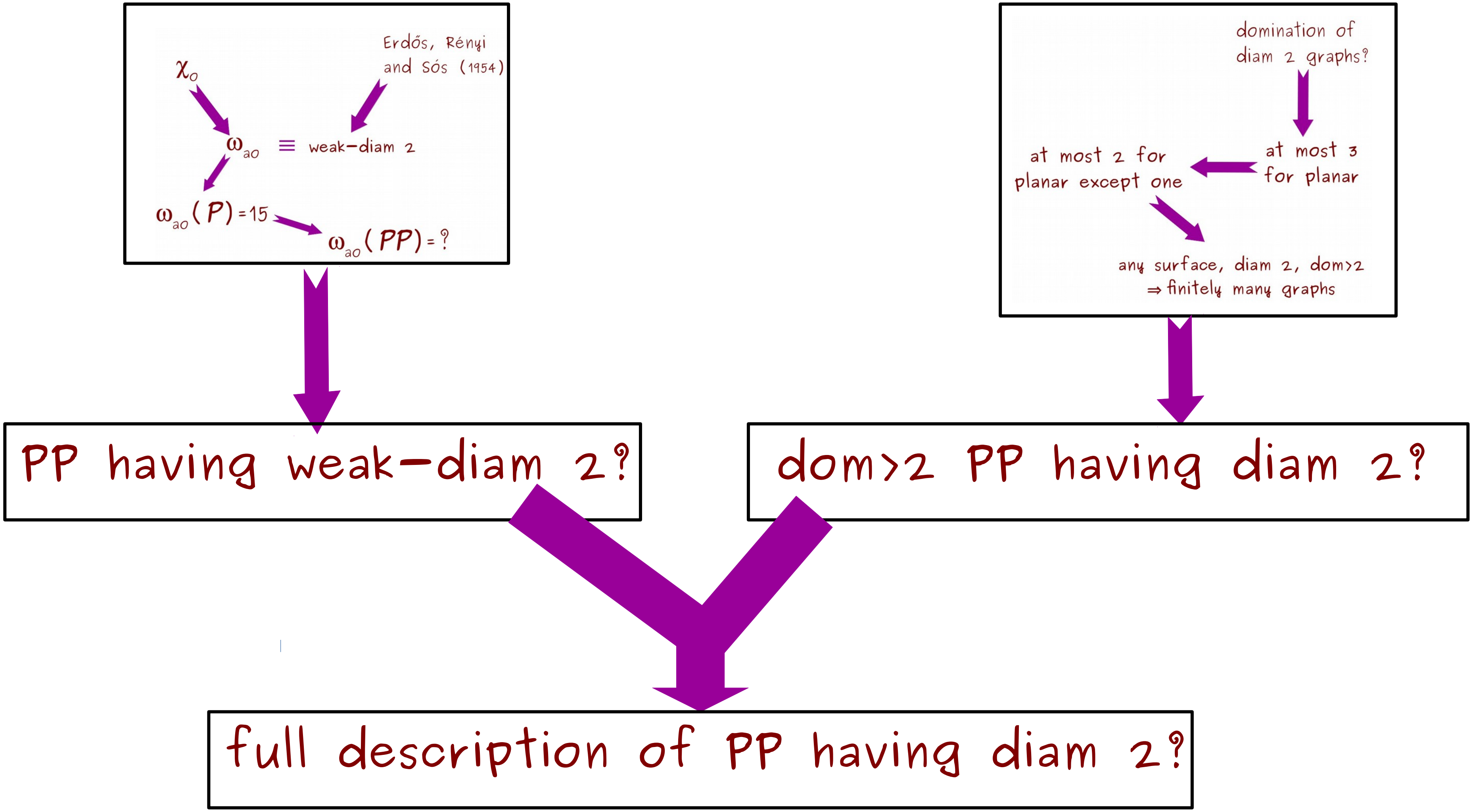


not even known for planar graphs!

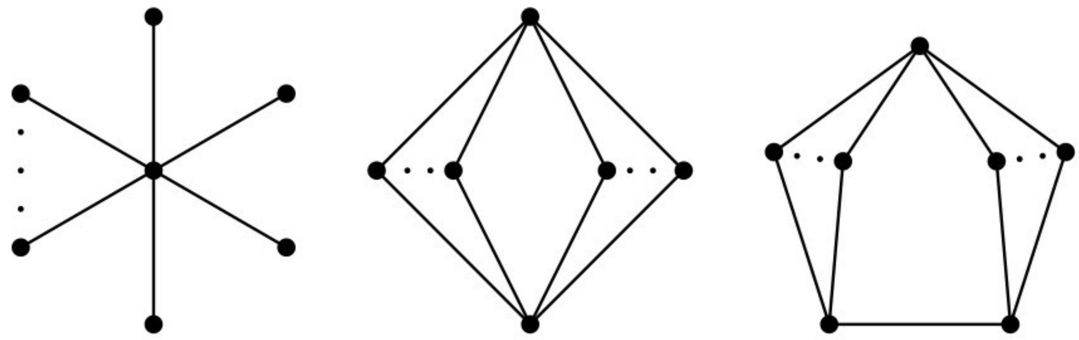


but known for triangle-free planar graphs!

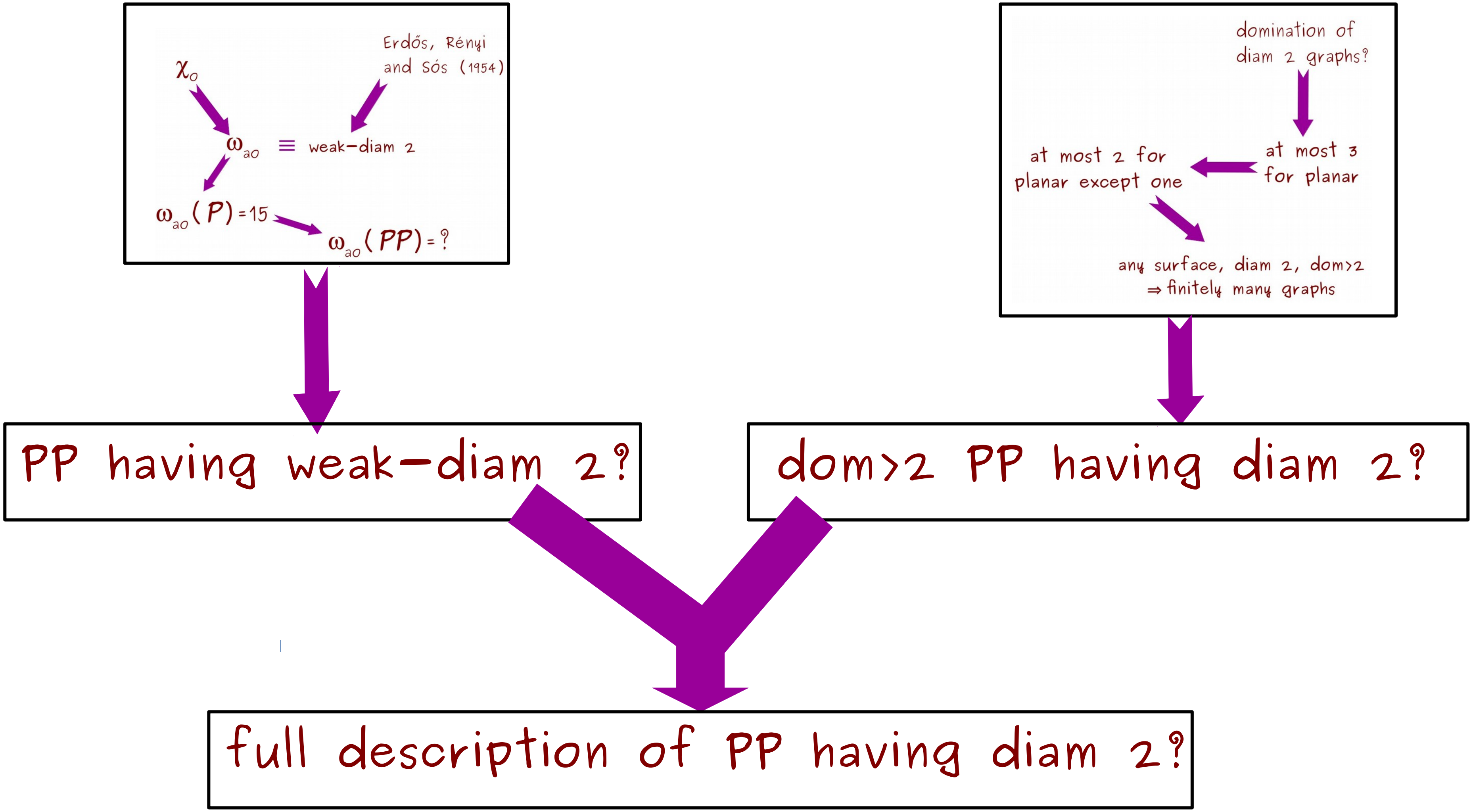
Known result

Theorem (Plesník 1975): Let G be Δ-free with diam 2. Then G is planar iff

$G \cong$



(a) $\mathcal{P}_1$ (b) $\mathcal{P}_2$ (c) $\mathcal{P}_3$



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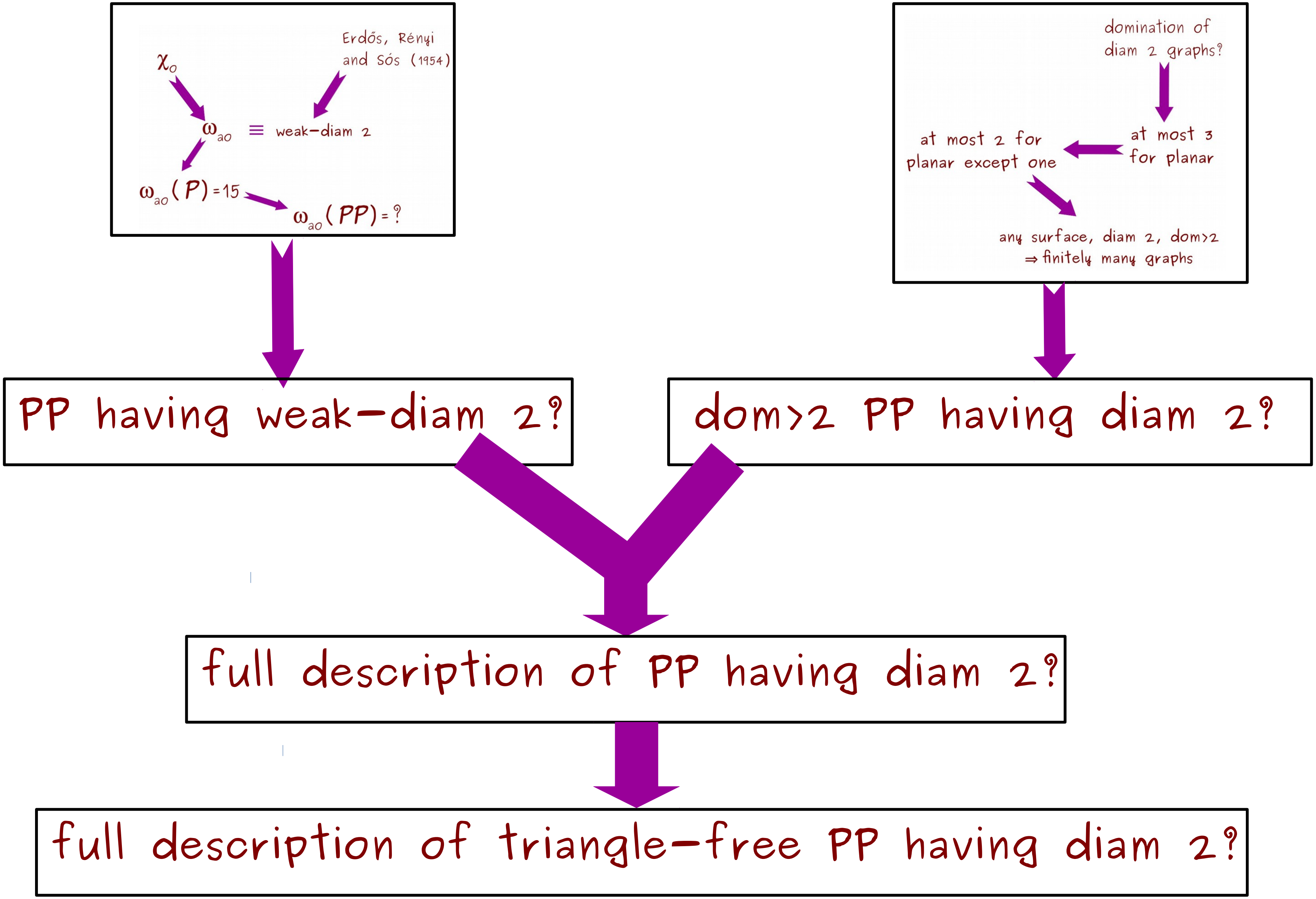

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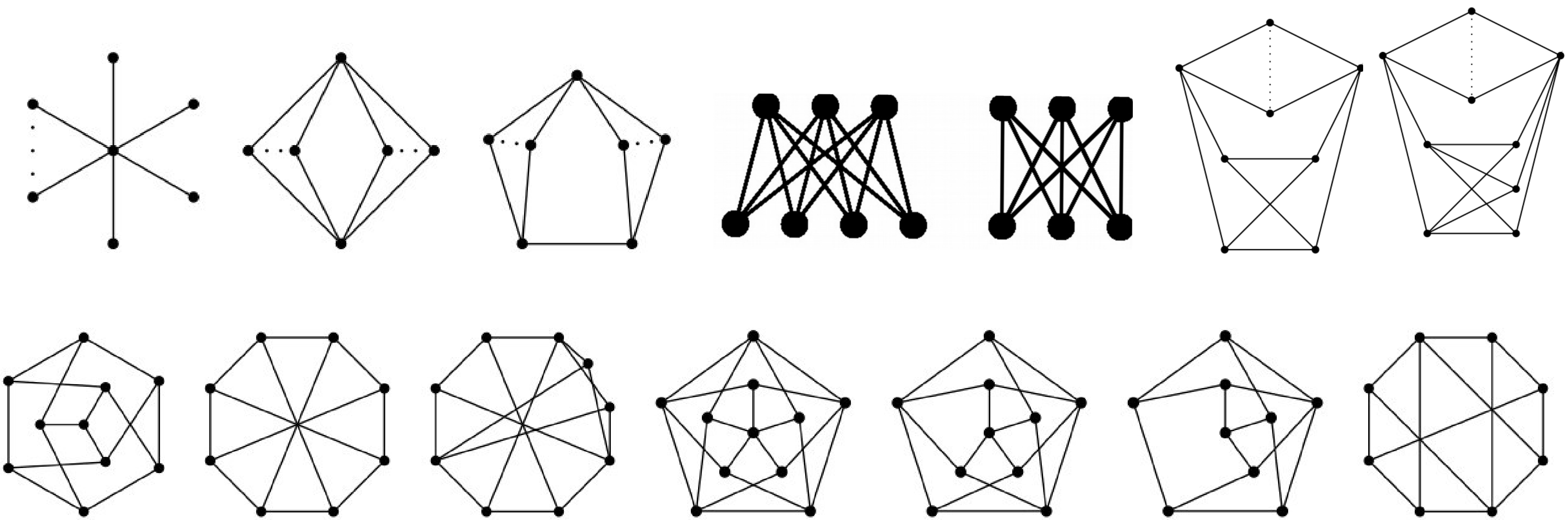
Our results

Theorem: Let G be Δ-free with diam 2. Then G is projective planar iff G is one of:

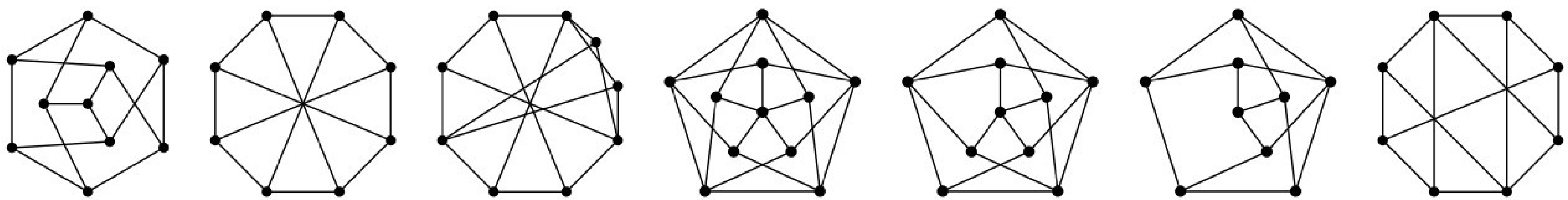


Implication

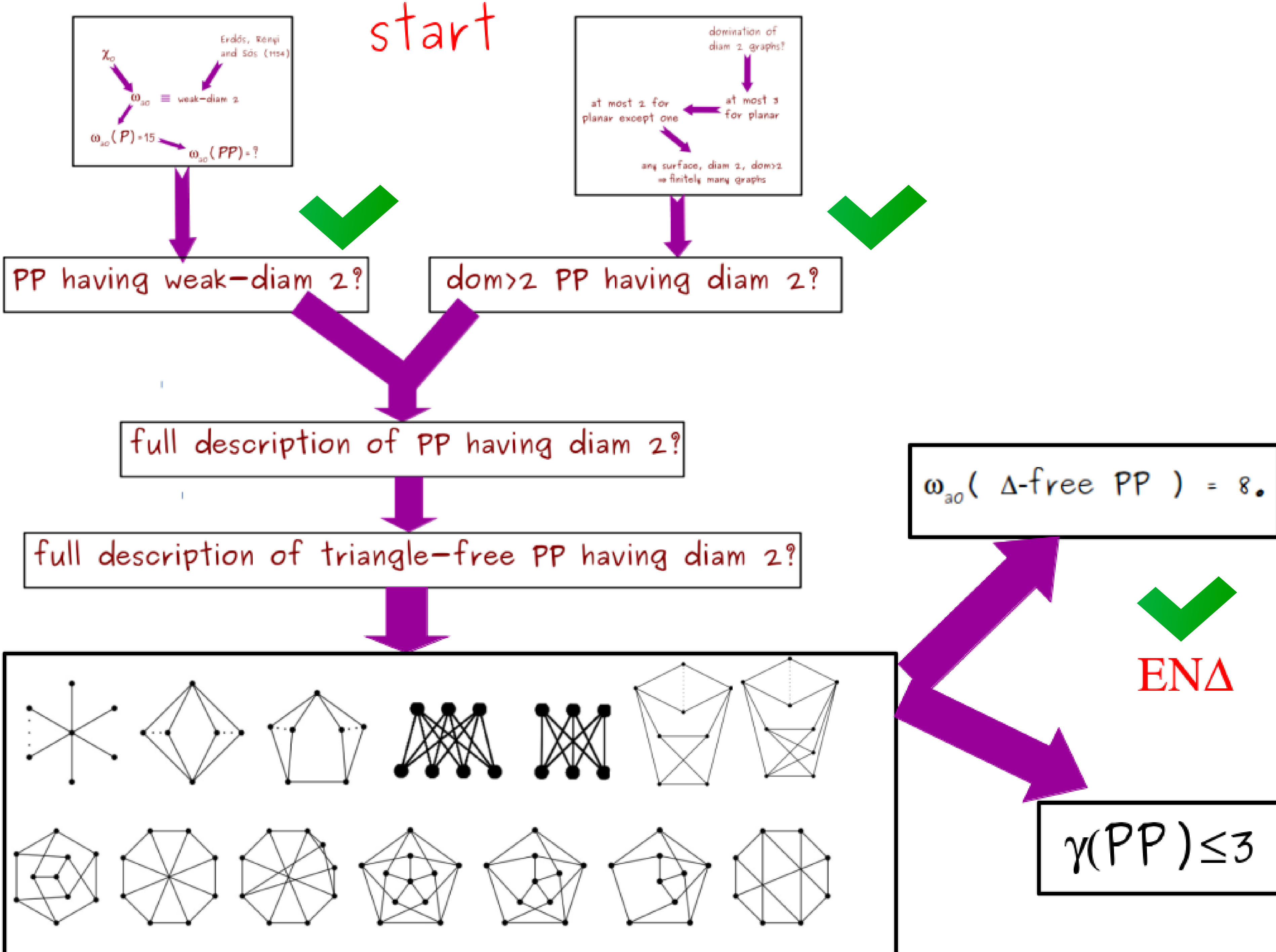
Theorem: $\omega_{ao}(\Delta\text{-free PP}) = 8.$

Implication

Theorem: Let G be Δ-free PPG with diam 2. Then domination number of G is 3 iff G is one of:



(a) (b) (c) (d) (e) (f) (g)



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ENΔ
$\gamma(PP)\leq 3$

time for a proof sketch...

Proof

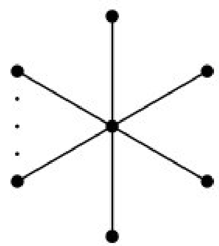
Observation: Let G be a Δ-free PP with diam 2. Then min degree of G is at most 3.

Euler's Formula

Proof

Observation: Let G be a Δ-free with diam 2 and min. degree = 1. Then G is PP iff

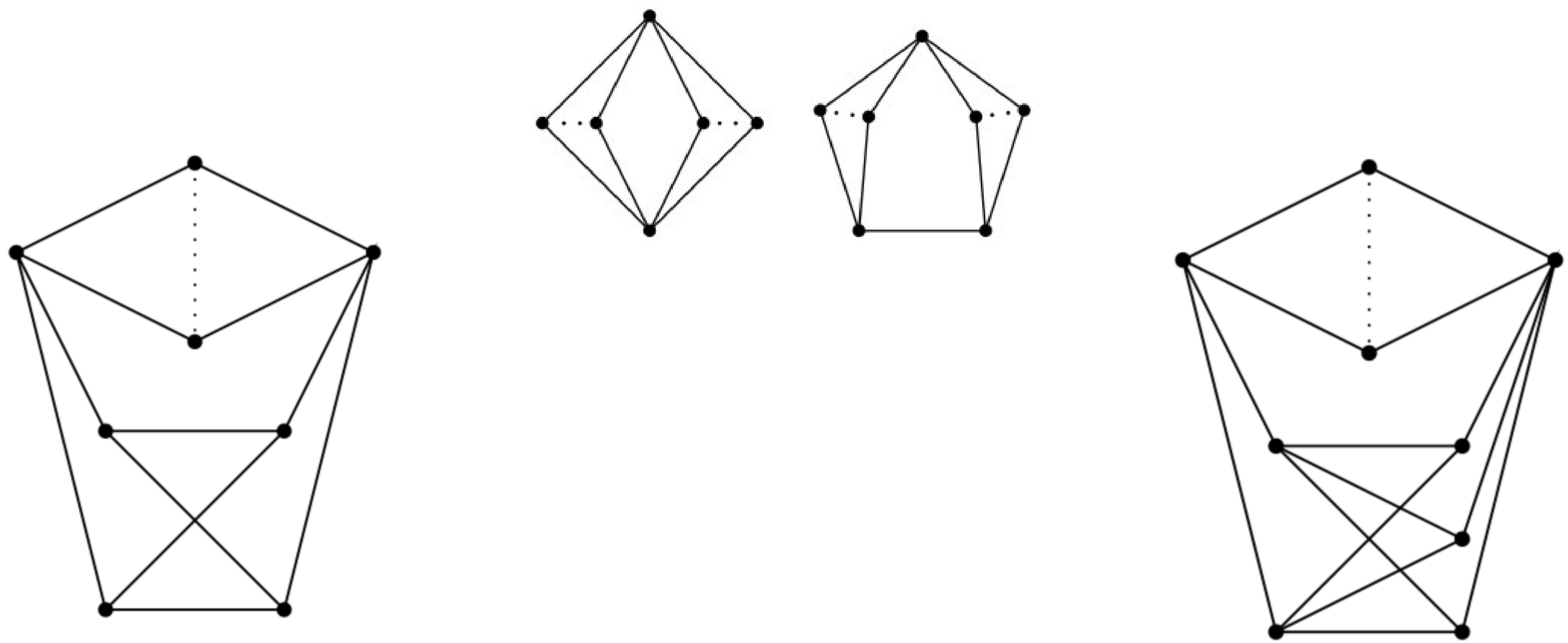
$G \cong$



Proof

Lemma: Let G be a Δ-free with diam 2 and min. degree = 2. Then G is PP iff

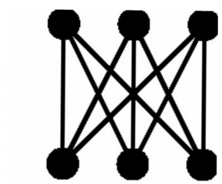
$G \cong$

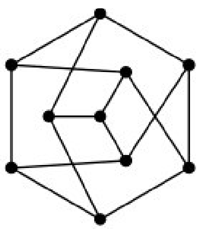


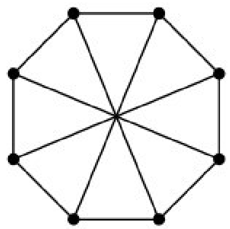
Proof

Lemma: Let G be 3-regular Δ-free with diam 2.
Then G is PP iff

$G \cong$



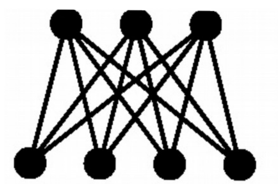


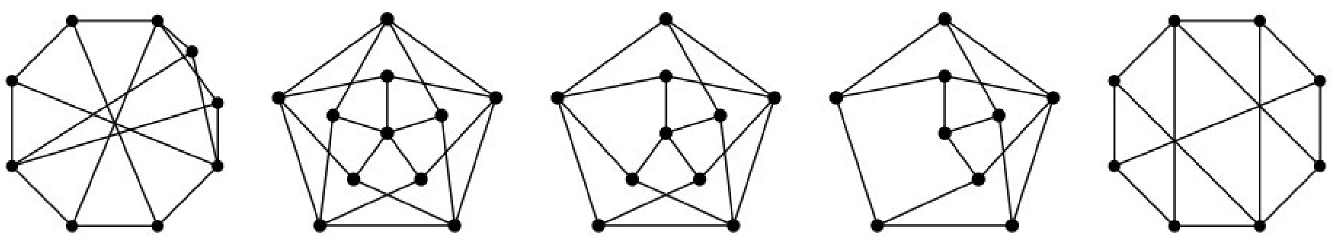


Proof

Lemma: Let G be Δ-free with diam 2, min degree =3 , max-degree ≥4. Then G is PP iff

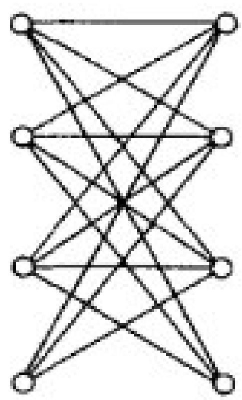
$G \cong$

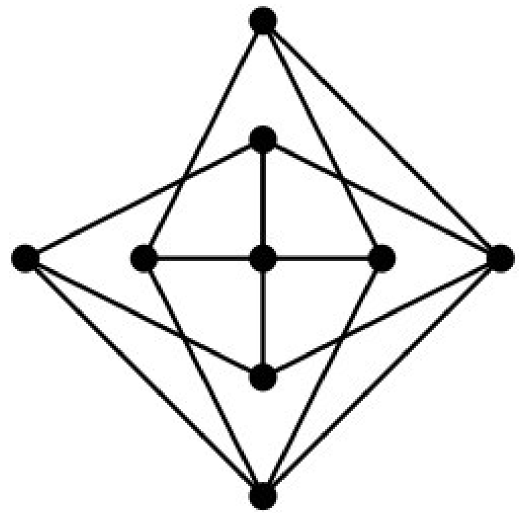


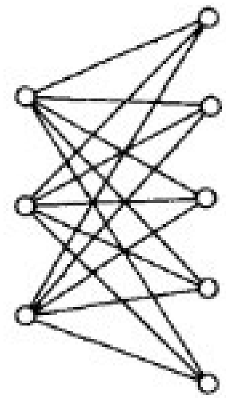


Results

Theorem: Let G be Δ-free with diam 2. Then G is PP iff G does not contain following as minor.







Thank you