On rectangle intersection graphs with stab number two

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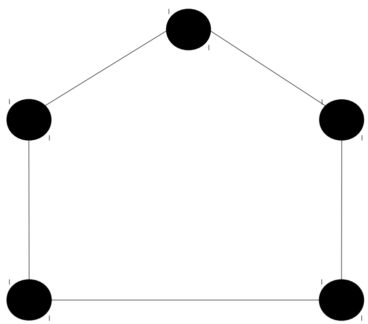
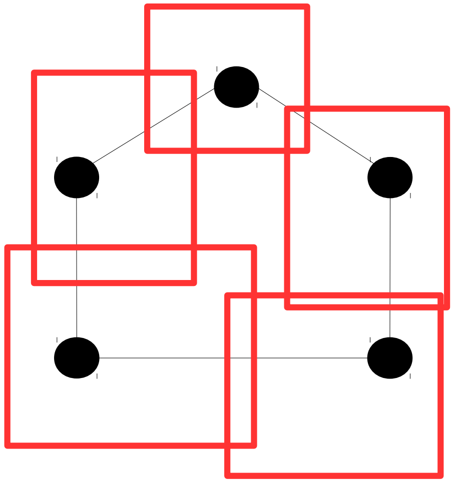
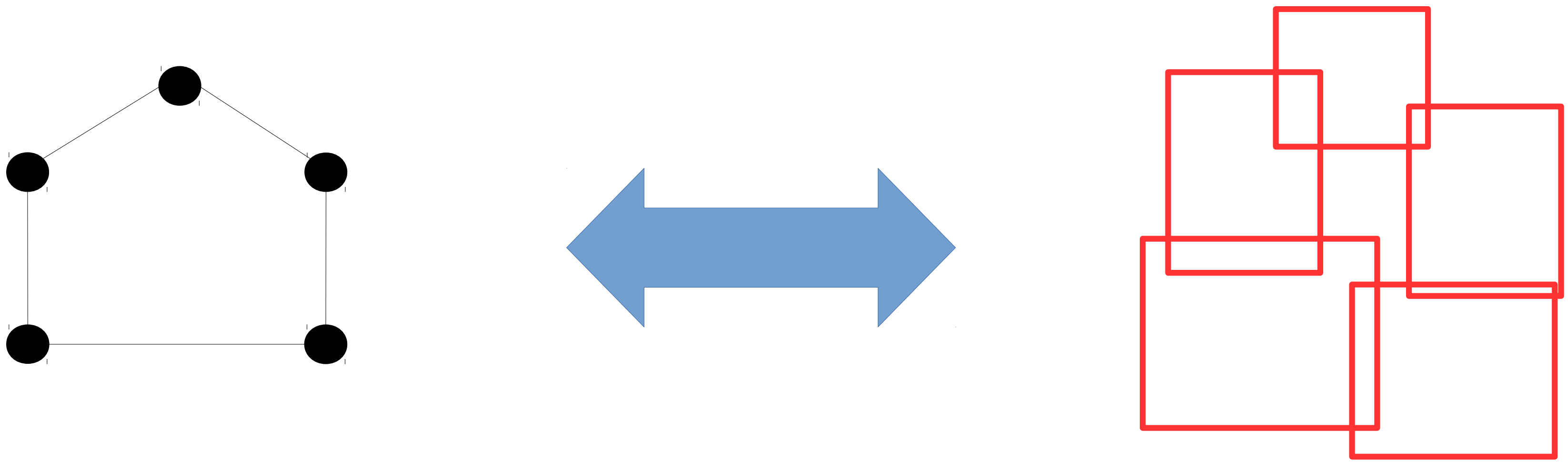
Rectangle intersection graph



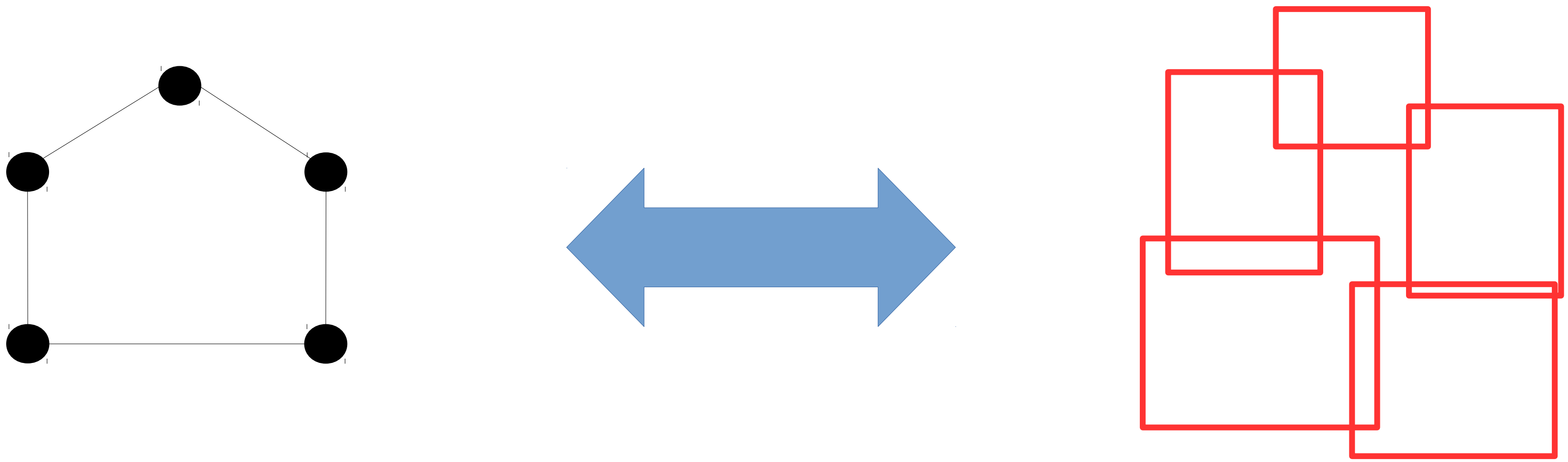
Rectangle intersection graph



Rectangle intersection graph



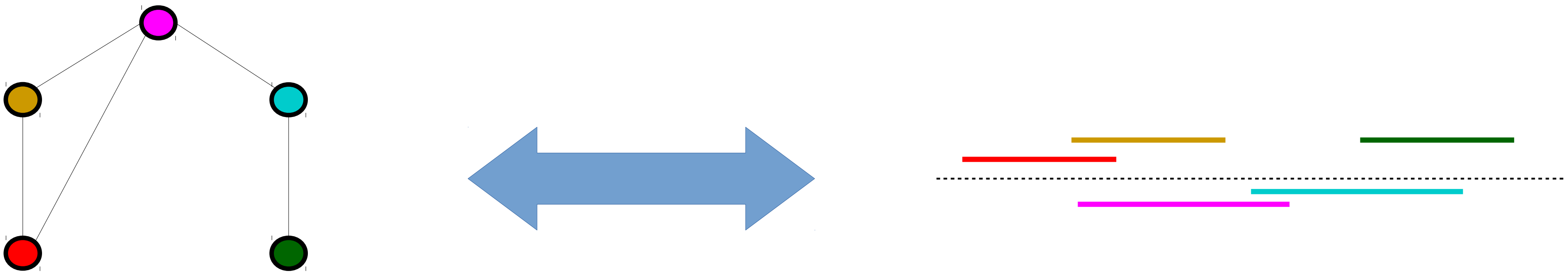
Rectangle intersection graph



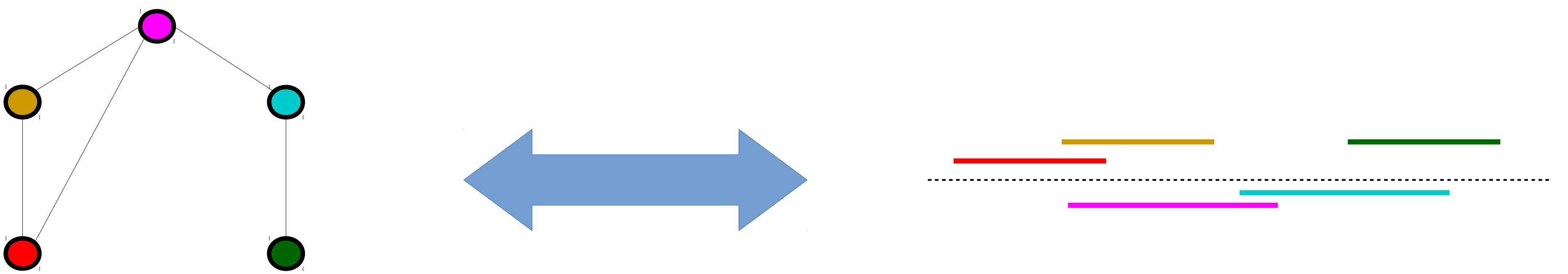
Generalisation of interval graphs

Interval graph

Interval graph



Interval graph



Lemma

A graph $G = (V, E)$ is a rectangle intersection graph if and only if there are interval graphs $I_1 = (V, E_1)$, $I_2 = (V, E_2)$ with $E = E_1 \cap E_2$

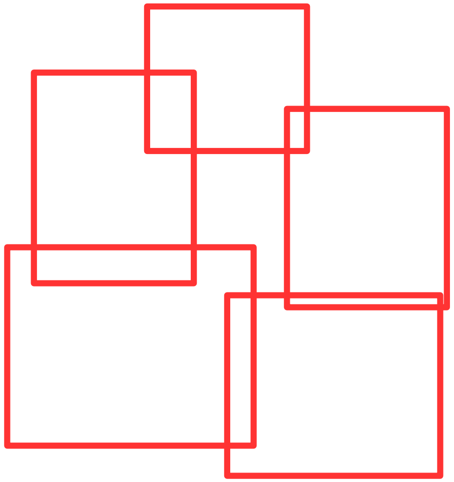
Rectangles make it difficult !

	Rectangle intersection graph	Interval graph
Recognition	NP-Hard	Polynomial
Coloring	NP-Hard	Polynomial
Clique number	Polynomial	Polynomial
Clique cover	NP-Hard	Polynomial
Maximum independent set	NP-Hard	Polynomial

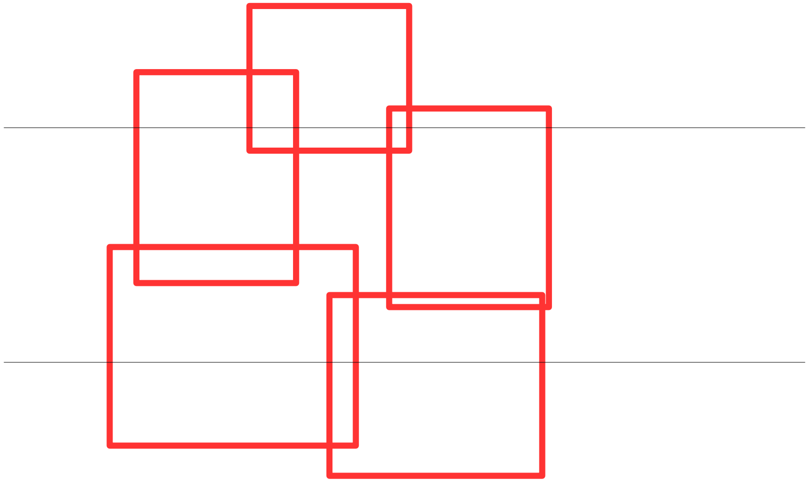
In this paper, we focus on a more restricted familiy called

rectangle intersection graphs with stab number two

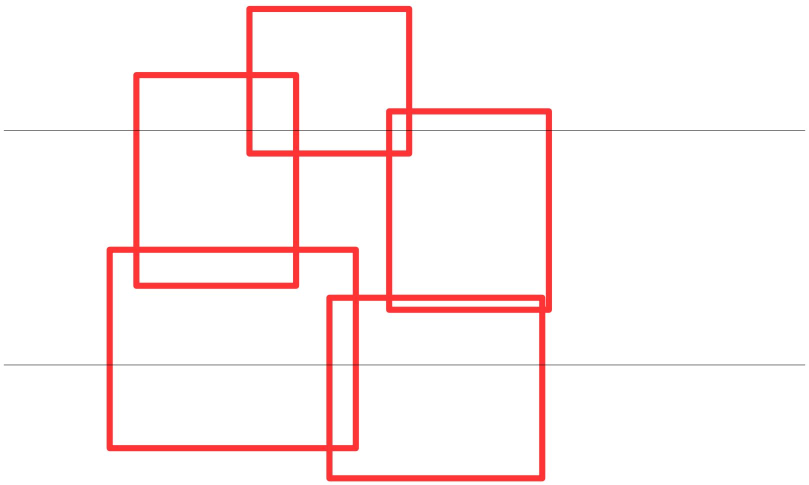
Definition



Definition

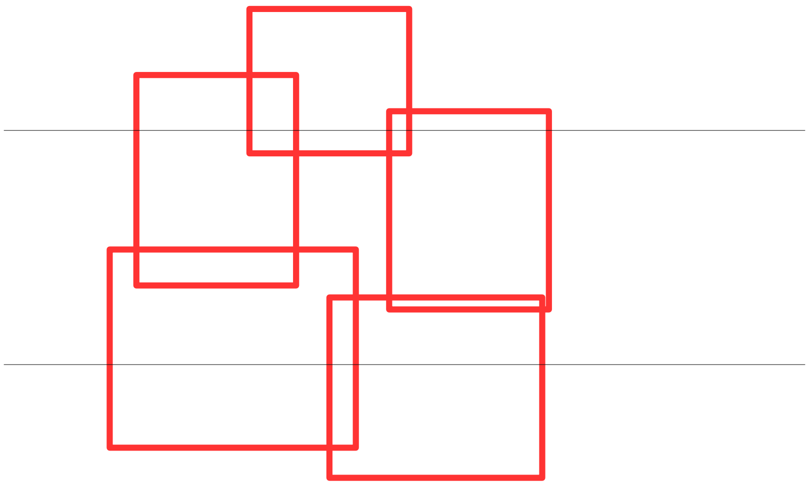


Definition



2-stabbed rectangle intersection representation

Definition



2-stabbed rectangle intersection representation

A graph G is a rectangle intersection graph with stab number two (2-SRIG) if

G has a 2-stabbed rectangle intersection representation

In this paper, we study the coloring problem on

2-SRIGs

Our Results on Coloring 2-SRIGs

Theorem 1

The CHROMATIC NUMBER problem is NP-Hard even on 2-SRIGs.

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Observe that

For any 2-SRIG H, $\chi(H) \leq 2\omega(H)$.

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The CHROMATIC NUMBER problem is NP-Hard even on 2-SRIGs.

Observe that

For any 2-SRIG H, $\chi(H) \leq 2\omega(H)$.

Theorem 2

There is a 2-SRIG H with $\chi(H) = \lceil \frac{3\omega(H)-1}{2} \rceil$.

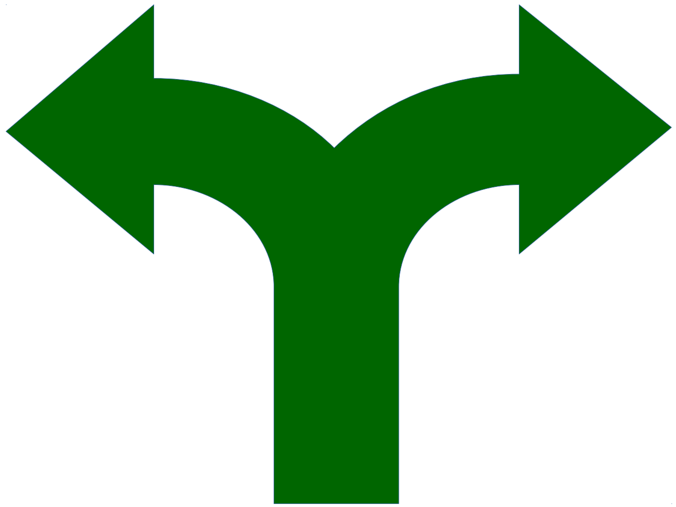
Proof Sketch

Theorem 1

Theorem 2

Proof Sketch

Theorem 1

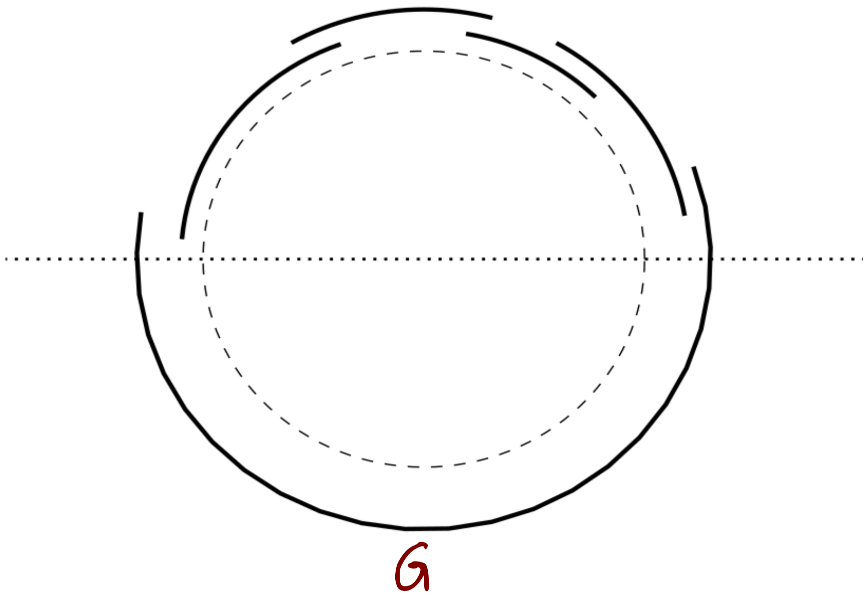


Theorem 2

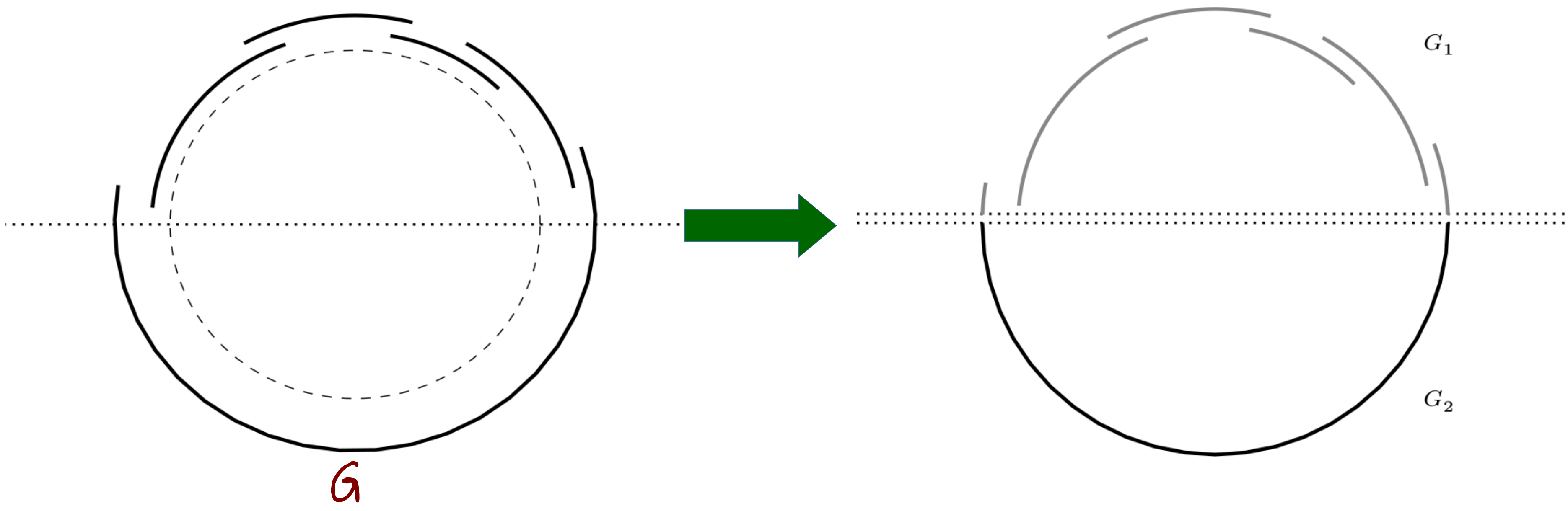
Lemma 3

For a circular-arc graph G and an integer k, there is a 2-SRIG H_k such that $\chi(G) \leq k$ if and only if $\chi(H_k) \leq k$.

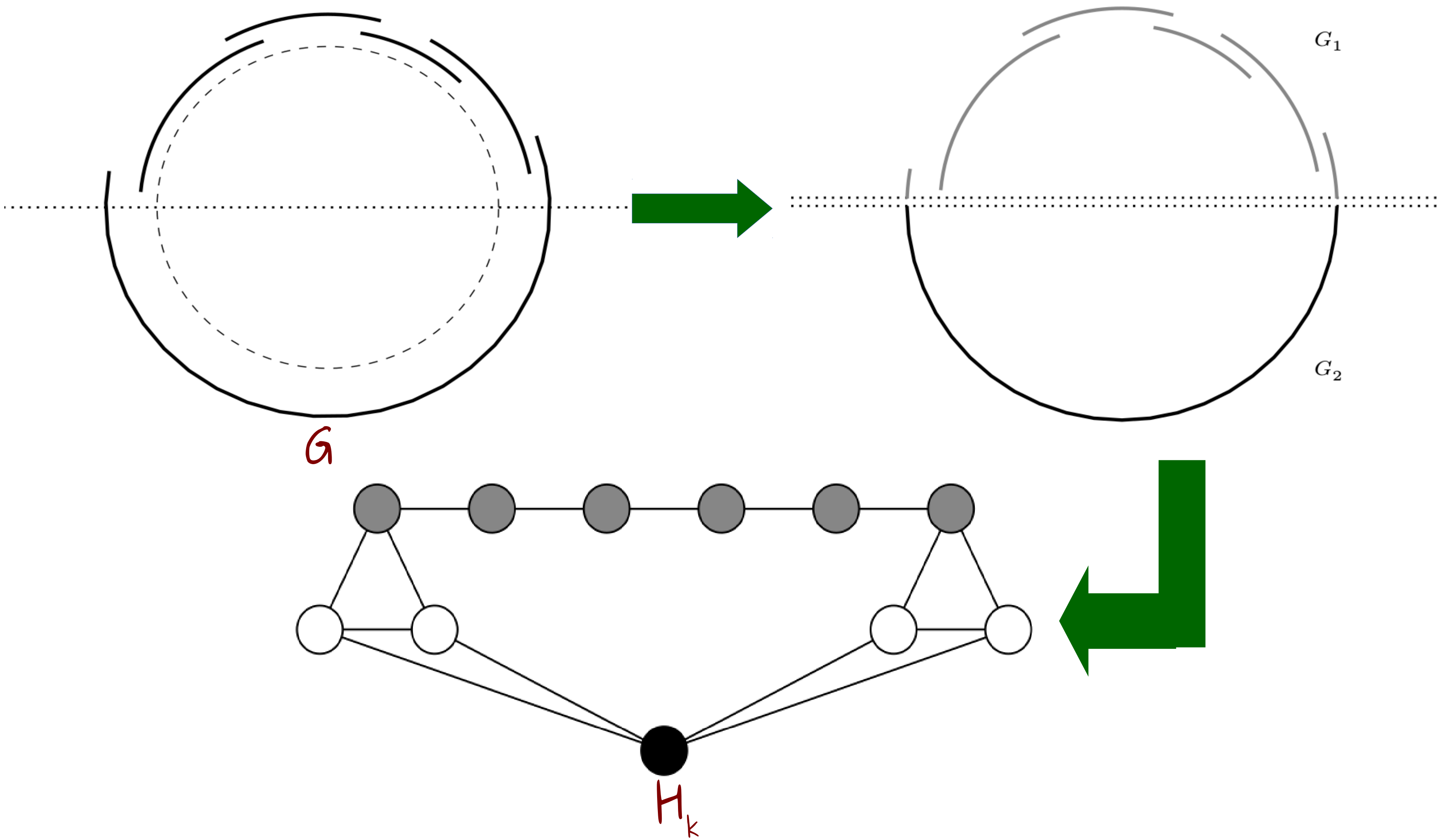
Proof Sketch: Lemma 3



Proof Sketch: Lemma 3



Proof Sketch: Lemma 3



In this paper, we study structural properties of subclasses of 2-SRIGs

Motivation

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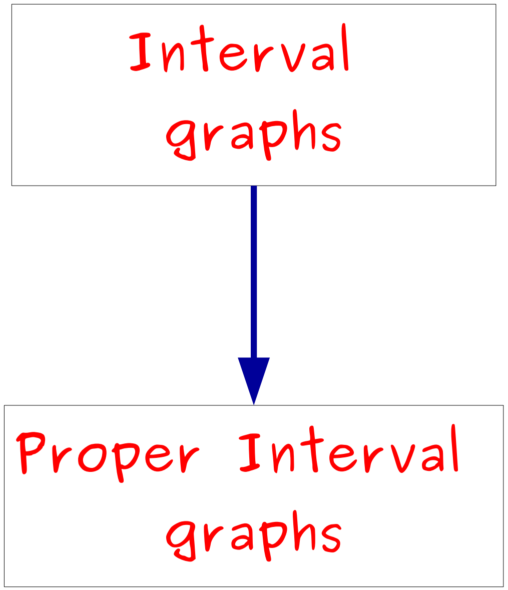
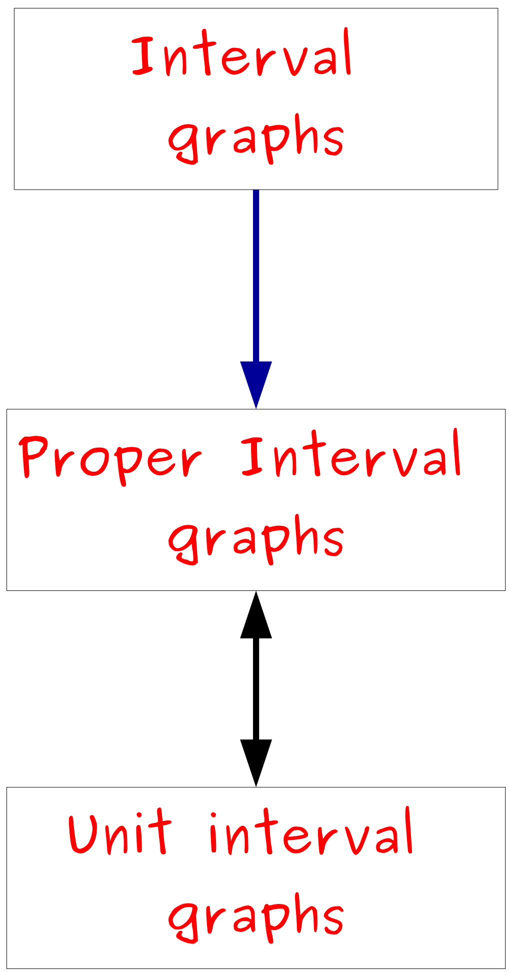


Intersection graphs of proper set of intervals.

Motivation



Intersection graphs of intervals of unit length.

Motivation

Define analogous graph classes for 2-SRIG and study the containment relationship among them.

Our Result

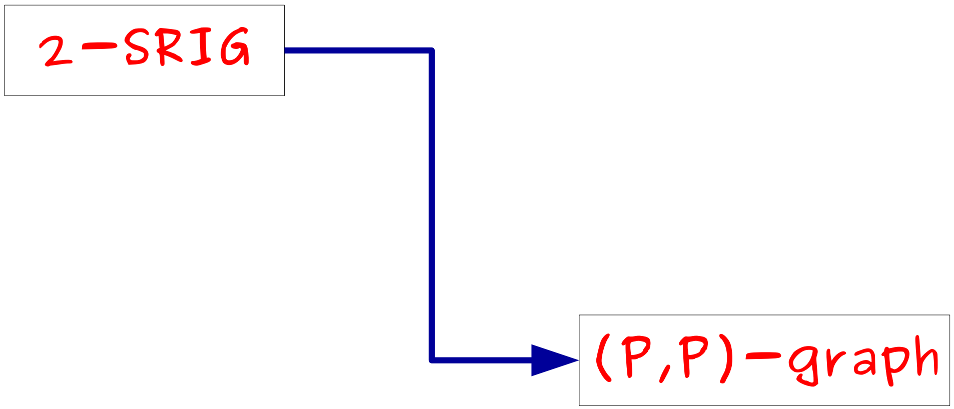
Theorem 3

2-SUIG $= (\mathcal{U},\mathcal{U})$-graphs $\subset$ $(\mathcal{P},\mathcal{U})$-graphs $= (\mathcal{P},\mathcal{P})$-graphs $\subset$ $(\mathcal{I},\mathcal{U})$-graphs $= (\mathcal{I},\mathcal{P})$-graphs $\subset$ 2-ESRIG = 2-SRIG.

Our Result

2-SRIG

Our Result



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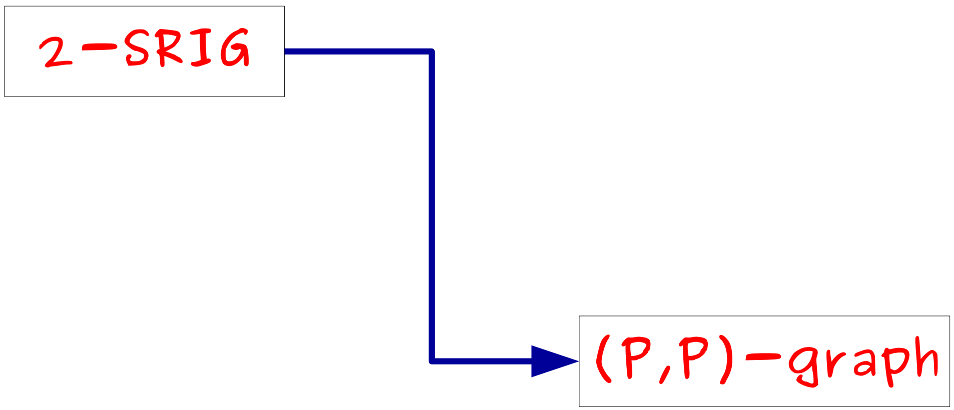
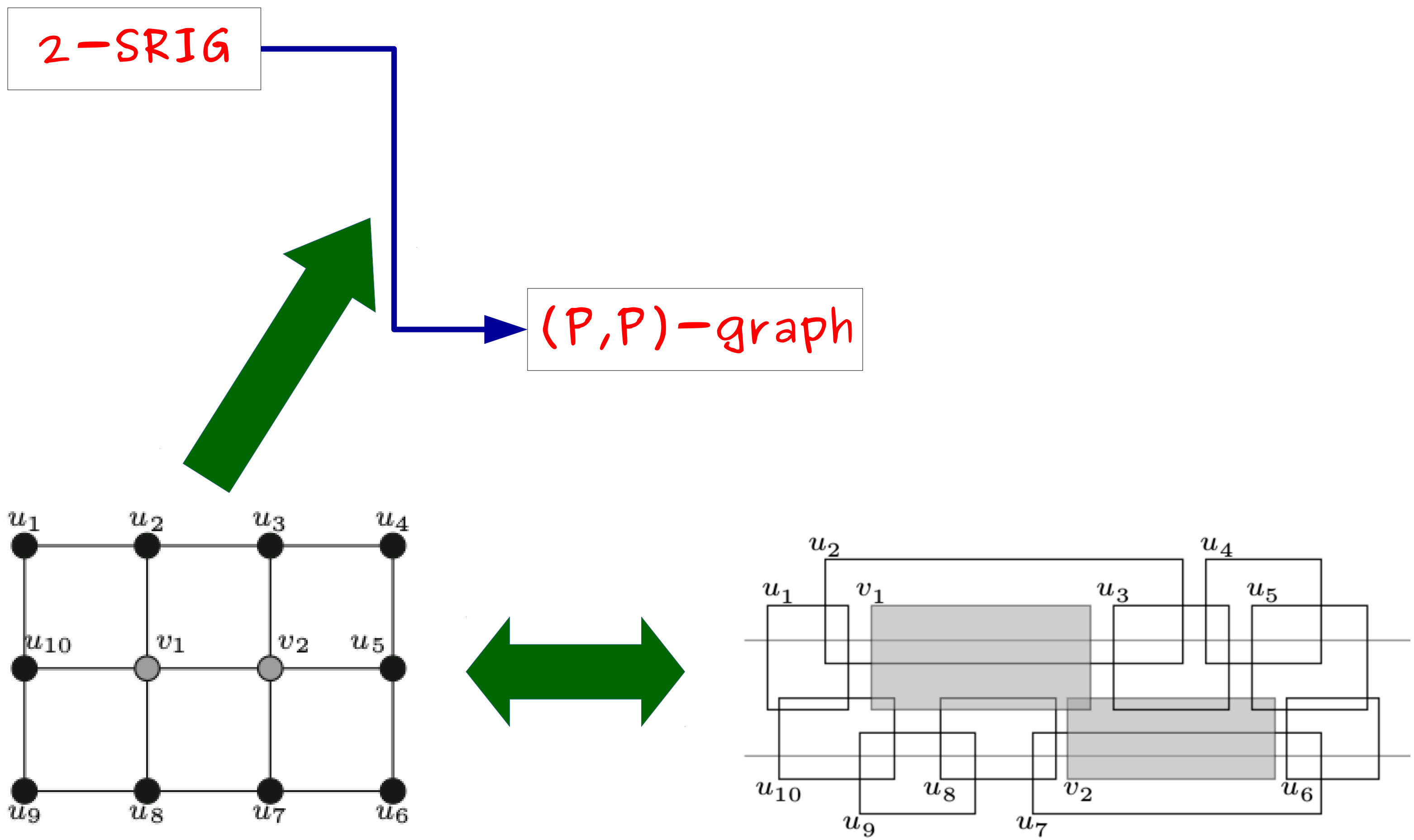


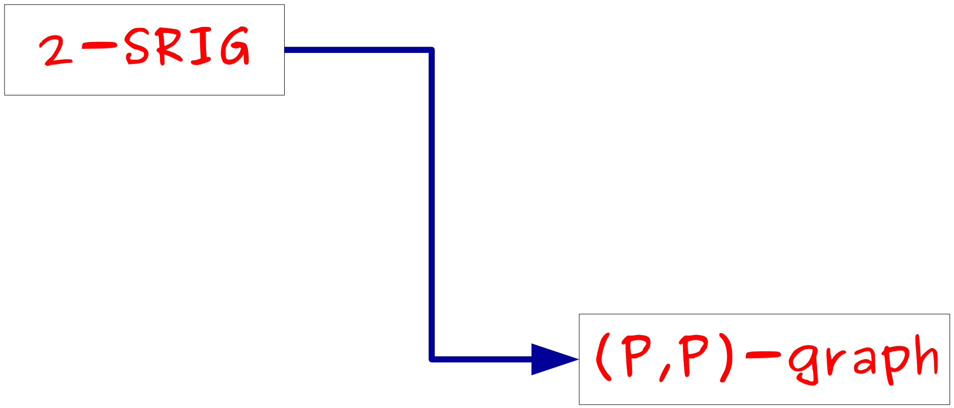
(P,P)-graph : Graphs having a 2-stabbed representation such that
(i) rectangles intersecting the **top stab line** gives a **proper set of intervals** and
(ii) rectangles intersecting the **bottom stab** line also gives a **proper set of intervals.**

Our Result



(3x4)-grid

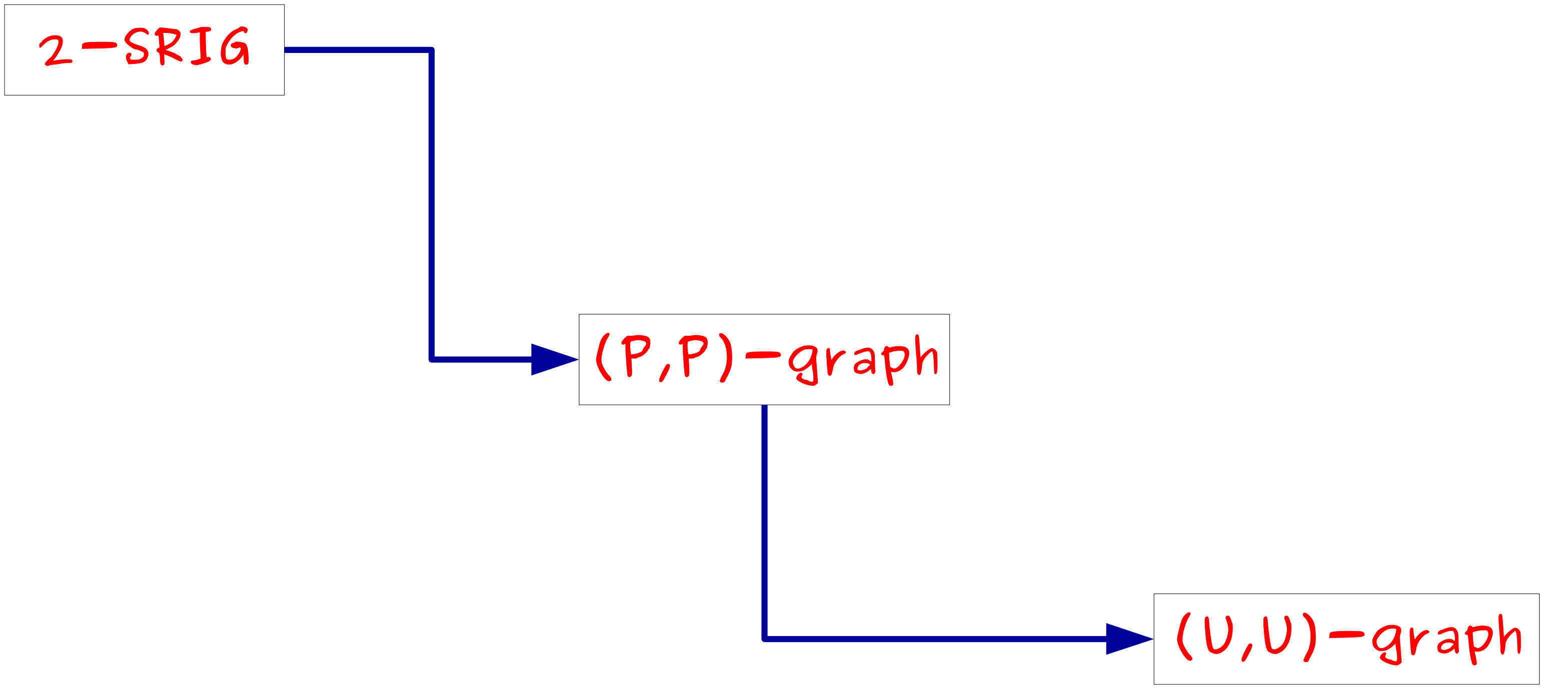
Our Result



Theorem 4

Let G be a triangle-free graph. There is an $O(|V(G)|)$-time algorithm to decide if G is a $(\mathcal{P},\mathcal{P})$-graph.

Our Result



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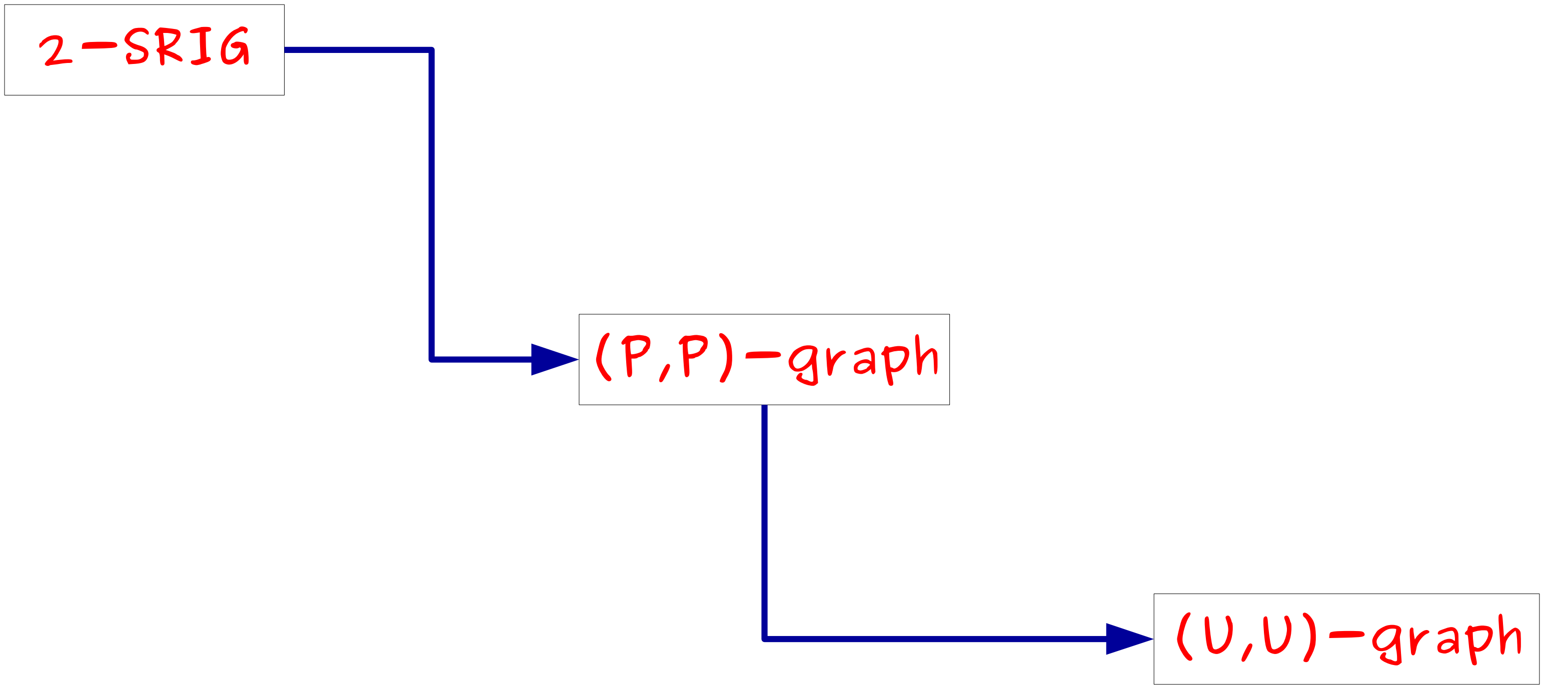
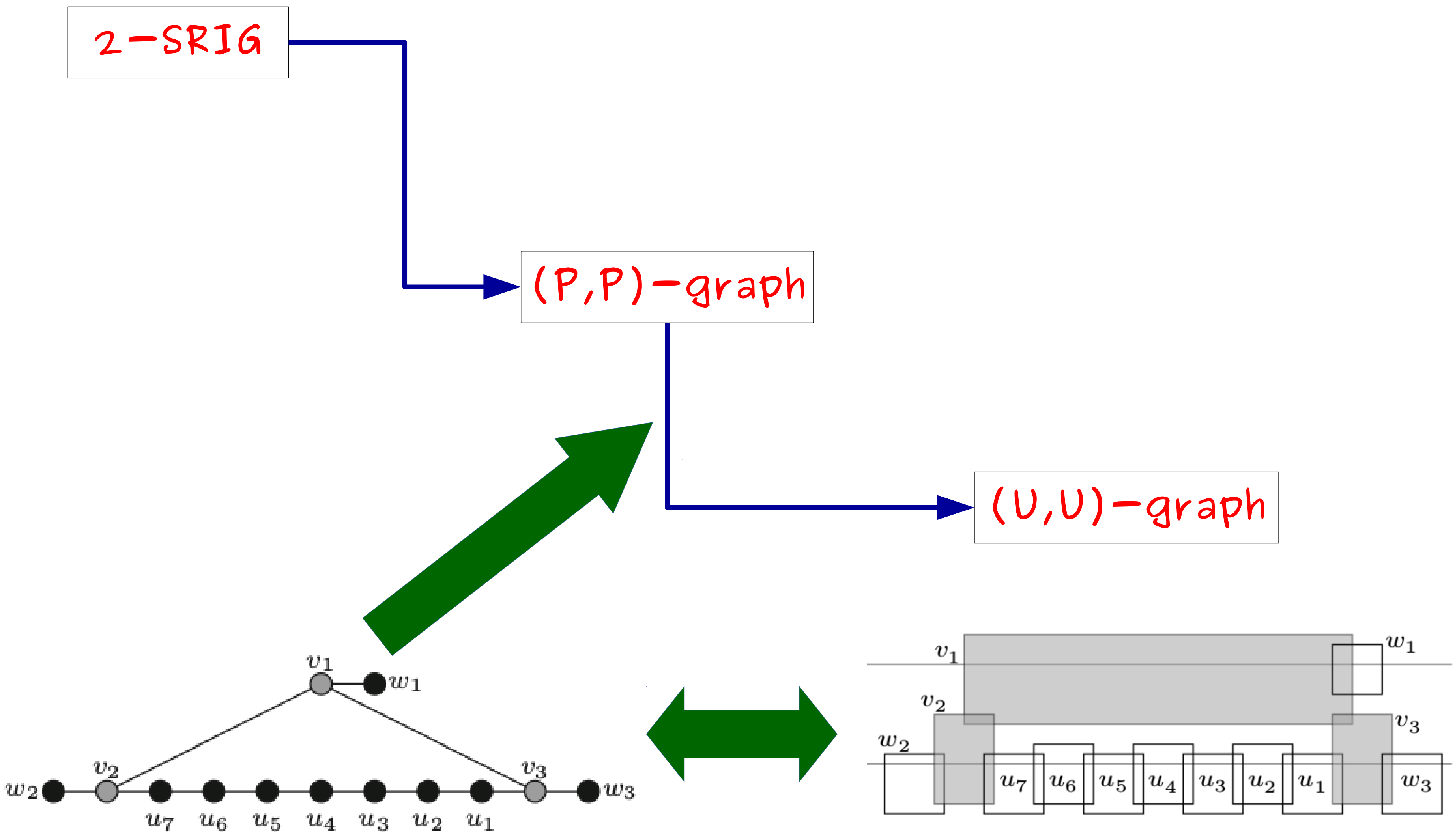
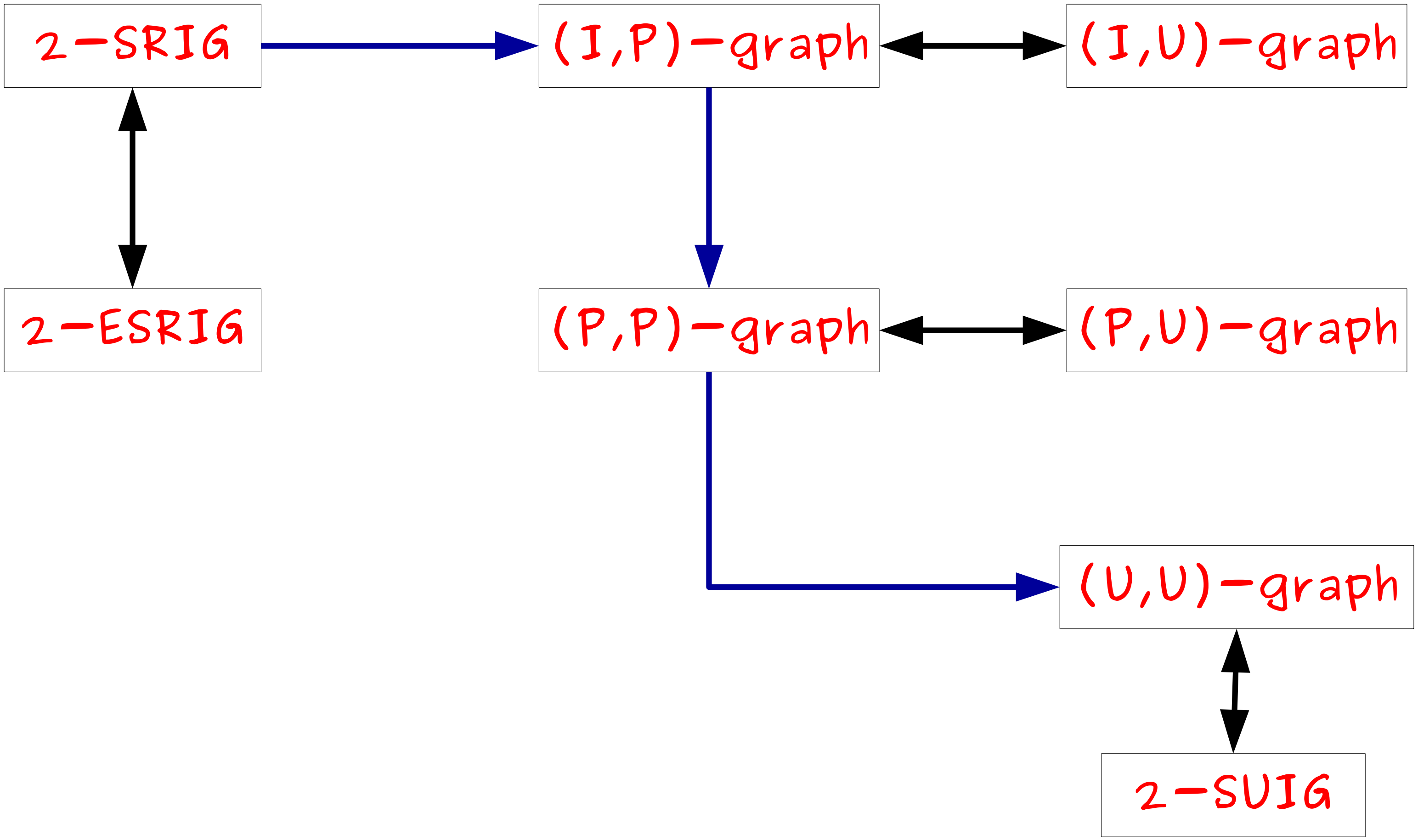


(U,U)-graph : Graphs having a 2-stabbed representation such that (i) rectangles intersecting the top stab line gives a set of unit intervals and
(ii) rectangles intersecting the bottom stab line also gives a set of unit intervals.

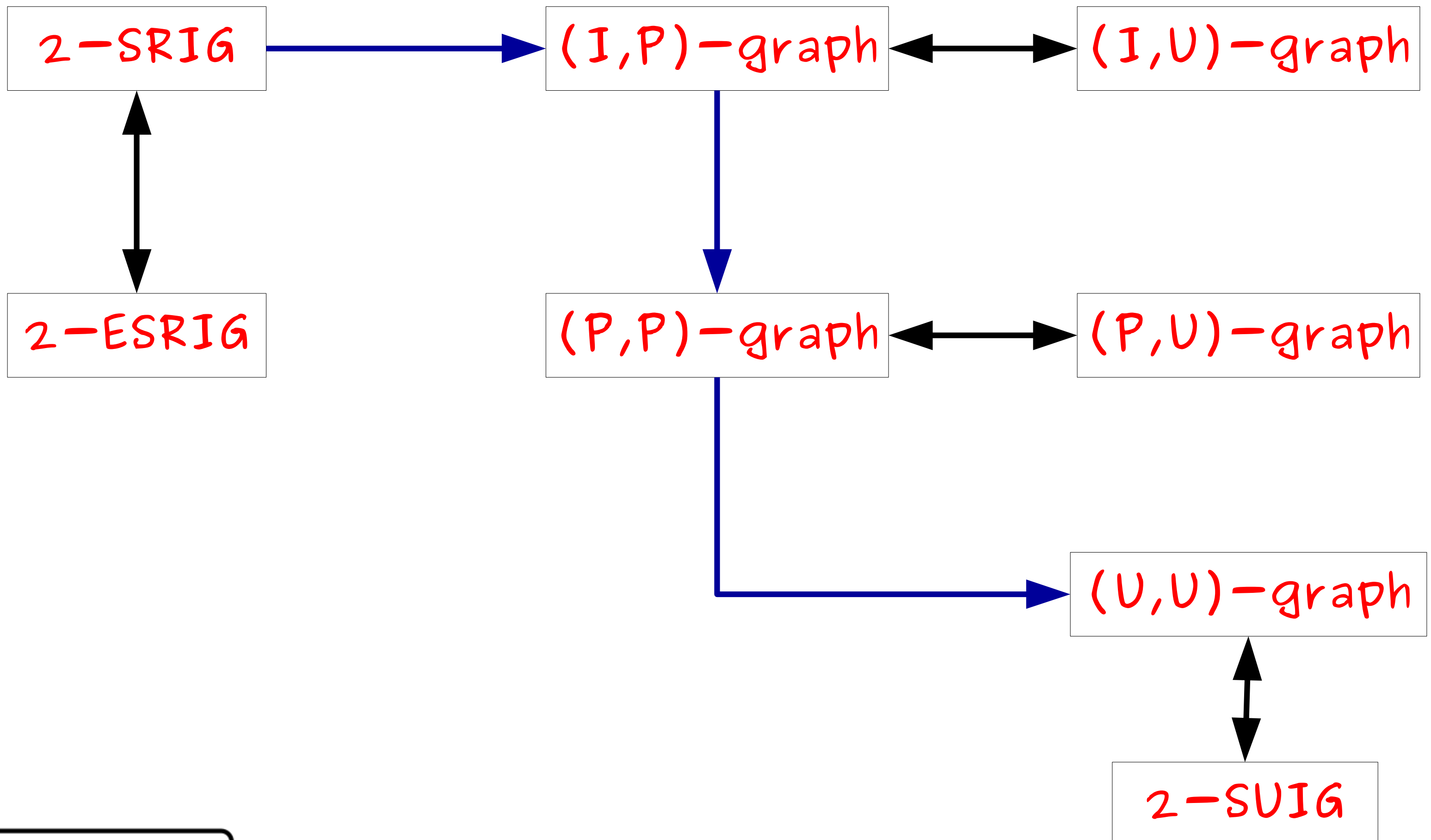
Our Result



Our Result


2-SRIG
(I,P)-graph
(I,U)-graph
2-ESRIG
(P,P)-graph
(P,U)-graph
(U,U)-graph
2-SUIG

Our Result



Theorem 3

2-SUIG $= (\mathcal{U},\mathcal{U})$-graphs $\subset$ $(\mathcal{P},\mathcal{U})$-graphs $= (\mathcal{P},\mathcal{P})$-graphs $\subset$ $(\mathcal{I},\mathcal{U})$-graphs $= (\mathcal{I},\mathcal{P})$-graphs $\subset$ 2-ESRIG = 2-SRIG.

Summary and Open Problems

We proved that

There is a 2-SRIG H such that $\chi(H) = \lceil \frac{3\omega(H)-1}{2} \rceil$?

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Question 1

Is it true that for any 2-SRIG H we have $\chi(H) \leq \frac{3\omega(H)}{2}$?

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We proved that

There is a 2-SRIG H such that $\chi(H) = \lceil \frac{3\omega(H)-1}{2} \rceil$?

Question 1

Is it true that for any 2-SRIG H we have $\chi(H) \leq \frac{3\omega(H)}{2}$?

Question 2

Is there a constant c such that for any rectangle intersection graph H we have $\chi(H) \leq c \cdot \omega(H)$?

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given a triangle-free graph G, there is an $O(|V(G)|)$-time algorithm to decide if G is a $(\mathcal{P}, \mathcal{P})$-graph.

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given a triangle-free graph G, there is an $O(|V(G)|)$-time algorithm to decide if G is a $(\mathcal{P},\mathcal{P})$-graph.

Question 3

Is there a polynomial time algorithm to recognise $(\mathcal{P},\mathcal{P})$-graphs ?

That's all folks.

Thank you for your attention.